7: Hillsborough

CALCULUS INDIVIDUAL TEST
REGIONAL COMPETITION - MARCH 14, 1992

NOTES: (1) "NOTA" means "None of the above."
(2) "DNE" means "Does not exist."

1. $\lim_{x \to -3} \frac{x^2 + 2x - 8}{x^2 + x - 6}$

A. -5 B. 4/5 C. ∞ D. DNE E. NOTA

2. Find the equation of the line that is normal to the graph of $f(x) = x^3$ at (2,8).

A. x - 12y = -94 B. x + 12y = 98 C. 12x - y = 16
D. 12x - y = 94 E. NOTA

3. $\lim_{x \to -1} \frac{x^{1/2} + 1}{x + 1}$

A. -1/2 B. 1/2 C. ∞ D. $-\infty$ E. NOTA

4. $\lim_{x \to 0^+} \frac{3}{1 + e^{-1/x}}$

A. 0 B. 3 C. ∞ D. DNE E. NOTA

5. If $f(t) = Arcsin\, t + Arccos\, t$, find $f'(\frac{\sqrt{2}}{2})$.

A. 0 B. $\frac{\sqrt{2}}{2}$ C. $\sqrt{2}$ D. $\frac{\pi}{2}$ E. NOTA

6. $\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$

A. -1/2 B. 1 C. $e^{-1/2}$ D. DNE E. NOTA

7. If $f(x) = |x|$ and $x \neq 0$, then $f'(x) =$

A. 1 B. $\frac{|x|}{x}$ C. $\frac{1}{2x}$ D. DNE E. NOTA

8. $\int_1^4 \frac{(x+1)^2}{\sqrt{x}}\,dx$

A. $\frac{15}{4} + \ln 16$ B. $\frac{27}{2} + \ln 4$ C. $\frac{356}{15}$ D. $\frac{110}{3}$ E. NOTA

9. $\int_{\frac{1}{e}}^{e} \frac{dx}{x(\ln x)^2}$

A. -2 B. 0 C. 1 D. 2 E. NOTA

10. $\int_0^{\sqrt{3}} \frac{dx}{1+x^2}$

A. $\frac{\pi}{6}$ B. $\ln 2$ C. $\frac{\pi}{3}$ D. $\ln 4$ E. NOTA

11. Find the volume of the solid generated by revolving about the Y-axis the region bounded by the graphs of

$$y = e^x,\ y = 0,\ x = 0,\ \textit{and}\ x = 1.$$

A. $\frac{e-1}{2}$ B. π C. 2π D. $\frac{\pi}{2}(e^2-1)$ E. NOTA

12. $\int \frac{dx}{x^2-4}$

A. $\ln|x^2-4|+C$ B. $\frac{1}{4}\ln\left|\frac{x+2}{x-2}\right|+C$ C. $\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

D. $\frac{2}{x+2}\cdot\ln|x-2|+C$ E. NOTA

13. $\int_0^{\frac{3}{2}} \frac{x\,dx}{\sqrt{1+2x}}$

A. 2/3 B. 4/3 C. 5/3 D. 20/3 E NOTA

14. Find $\lim_{n\to+\infty}\left(\left(\sin\frac{\pi}{n}+\sin\frac{2\pi}{n}+\ldots+\sin\frac{n\pi}{n}\right)\cdot\frac{\pi}{n}\right)$

A. 0 B. 1 C. 2 D. π E. NOTA

15. A particle P starts at A(6,0) and moves counterclockwise along the circle $x^2+y^2=36$ at the constant rate of 1 revolution per 12 sec. Find the rate at which the length of the segment AP is changing at the end of 4 sec. (t = 4) [the answer will be in linear units per second].

A. $6\pi\sqrt{3}$ B. $6\sqrt{3}$ C. π D. $\frac{\pi}{2}$ E. NOTA

16. Find the length of the graph of $f(x)=\frac{2}{3}x^{3/2}$ from (1, 2/3) to (4, 16/3).

A. $\frac{14}{3}$ B. $\sqrt{31}$ C. $\frac{2}{3}(5^{3/2}-1)$ D. $\frac{2}{3}(5^{3/2}-2^{3/2})$ E. NOTA

17. Given the set of (Roman) numbered statements:
I. If f is continuous on [a,b], then it is integrable on [a,b].
II. If f is continuous on [a,b], then it is differentiable on [a,b].
III. If f is integrable on [a,b], then it is continuous on [a,b].
IV. If f is differentiable on [a,b], then it is continuous on [a,b].
V. If f is differentiable on [a,b], then it is integrable on [a,b].
VI. If f is integrable on [a,b], then it is differentiable on [a,b].

Which of the following subsets indicate all true statements?

A. {I, II, IV} B. {I, III, IV} C. {III, IV, VI}

D. {I, IV, V} E. NOTA

18. If $f(x) = x$, $g(x) = \cos x + x$, *and* $0 \le x \le 2\pi$, find the area between the graphs of the two functions on the given interval.

A. 0 B. 2 C. 3 D. 4 E. NOTA

19. $f(x) = x^4 - 4x + 1$

A. Has a relative maximum at x = 1 and has a point of inflection at (0,1).
B. Has a relative minimum at x = 1 and has a point of inflection at (0,1).
C. Has a relative maximum at x = 1 and has no inflection point.
D. Has a relative minimum at x = 1 and has no inflection point.
E. NOTA

20. At 1 pm A is 34 miles due east of B and traveling at a constant speed of 10 mph due west. B is traveling due south at a constant speed of 6 mph. In how many hours is the distance between A and B minimum?

A. 2.5 B. $\frac{170}{53}$ C. 3.4 D. 8.5 E. NOTA

21. Find the area of the region bounded by

$$y = \sec\frac{\pi}{8}x,\ y = 0,\ x = 0,\ \textit{and}\ x = 2.$$

A. $\frac{8}{\pi}\ln(\sqrt{2}+1)$ B. $\frac{8}{\pi}\ln\sqrt{2}$ C. $\frac{8\sqrt{2}}{\pi}$ D. $\sqrt{2}$ E. NOTA

22. A body moves along a horizontal line according to the law

$$\text{position} = f(t) = t^3 - 15t^2 + 63t.$$

The speed of the body is decreasing when

A. $3 < t < 7$ B. $t < 3$ or $t > 7$ C. $t < 3$ or $5 < t < 7$

D. $3 < t < 5$ or $t > 7$ E. NOTA

23. Water is running out of an inverted conical funnel at the rate of 1 mL/s (1 mL = 1 cm^3). If the diameter of the base (top) of the funnel is 16 cm and the altitude is 8 cm, at what rate is the water level dropping when it is 2 cm from the top? [The solution is in cm/s].

A. $\frac{1}{36\pi}$ B. $\frac{1}{4\pi}$ C. $\frac{8\pi}{3}$ D. 72π E. NOTA

24. If $f'(x) = 3x^{1/2} + 7$ *and* $f(4) = 48$, *find* $f(x)$.

A. $f(x) = \frac{3}{2}x^{-1/2} + \frac{189}{4}$ B. $f(x) = 2x^{3/2} + 7x + 4$

C. $f(x) = 2x^{3/2} + 7x - 4$ D. $\frac{9}{2}x^{3/2} + 7x - 16$ E. NOTA

25. Find the approximate change in the volume of a cube of side s cm when the sides are increased by 0.4%.

A. 0.012 s^3 B. 0.12 s^3 C. 0.64 s^3 D. 1.012 s^3 E. NOTA

26. If $F(x) = \int_3^{x^2} t(t^2+2)^{1/2}\,dt$, $D_x F(x) =$

A. $x(x^2+2)^{1/2}$ B. $x^2(x^4+2)^{1/2}$ C. $2x^3(x^4+2)^{1/2}$

D. $\frac{1}{3}(x^4+2)^{3/2}$ E. NOTA

27. Find the average value of $f(x) = x^2$ over $[1,4]$.

A. 5 1/4 B. 7 C. 7 1/2 D. 21 E. NOTA

28. *If* $f(x) = \ln x$ *and* $g(x) = x^2 + 2x$, *then* $f \circ g(x) =$

A. $\ln x + \ln(x+2)$ B. $3\ln x + \ln 2$ C. $(\ln x)^2 + \ln x^2$

D. $\ln x^4$ E. NOTA

29. If water weighs 62 lb/ft^3, find the work required to pump all the water from a full cylindrical container to its top. The dimensions of the cylinder are: diameter 3 ft. and height 4 ft. [The answer will be in ft-lbs].

A. 18π B. 576π C. 1116π D. 4464π E. NOTA

30. If $y = x^x$, find $D_x y$ [assuming $x > 1$].

A. 1 B. x C. x^x D. $x^x(1 + \ln x)$ E. NOTA