MAR 9/91 FAMAT REGIONAL COMPETITION CALCULUS INDIVIDUAL TEST

1. What is the domain of the function : $f(x) = \sqrt{(x^2-1)(x-4)}$

A. all real numbers B. $[-1,\infty)$ C. $(-\infty,-1] \cup [4,\infty)$

D. $[-1,1] \cup [4,\infty)$ E. NOTA

2. What is the range of the derivative of: $y = \dfrac{x^3}{1+x^3}$

A. all real numbers except 0 B. $(0,+\infty)$

C. $(-\infty,-1) \cup (-1,+\infty)$ D. $[0,+\infty)$

E. NOTA

3. For what values of x is the function $f(x) = \dfrac{\sqrt{9-x^2}}{x^4-16}$ continuous?

A. $x=\pm2$ and $x=\pm3$ B. $(-\infty,-3) \cup (3,\infty)$ C. $[-3,3]$

D. $(-\infty,-3] \cup (-2,2) \cup [3,+\infty)$ E. NOTA

4. If $f(x) = \dfrac{g(x)}{x^2-10x+25}$ and it is known that $\lim_{x\to 5} g(x) = 0$ which of the following statements is always true of f(x)?

A. $\lim_{x\to 5} f(x) = 0$ B. $\lim_{x\to 5} f(x)$ does not exist

C. $\lim_{x\to 5} f(x)$ exists but may equal something other than zero.

D. $\lim_{x\to 5} f(x)$ may or may not exist.

E. NOTA

5. What function is the inverse of $f(x) = \ln(x^2-9)$?

A. $f^{-1} = \sqrt{e^x+9}$ B. $f^{-1} = \pm\sqrt{e^x+9}$ C. $f^{-1} = e^{x^2-9}$

D. $f^{-1} = \pm\sqrt{e^x}+9$ E. NOTA

6. If $f(x) < 1$ and $f'(x) < 0$ for all x's , then

$$g(x) = \frac{1}{f(x) - 1}$$

A. g(x) is increasing over its entire domain.
B. g(x) is decreasing over its entire domain.
C. g(x) is increasing for part of its domain and decreasing for the rest of its domain.
D. g(x) does not exist and therefor is neither increasing nor decreasing.
E. NOTA

7. Find $\lim_{x \to 0} \frac{\sqrt{6 + 2x} - \sqrt{6 + x^2}}{\sqrt{3 + 4x} - \sqrt{3 - x^3}}$

A. 1 B. $\frac{3\sqrt{2}}{4}$ C. $\frac{\sqrt{2}}{4}$ D. $\sqrt{2}$ E. NOTA

8. Evaluate $\lim_{x \to 0} \frac{2x}{\arctan x}$

A. 0 B. 1 C. $\frac{\pi}{2}$ D. 2 E. NOTA

9. Find $\frac{dy}{dx}$ if $y = e^{-\ln(\frac{1}{x})} + \ln(\frac{1}{e^{2x}})$

A. 1 B. e C. $2e^x$ D. ln x E. NOTA

10. If $y = x - x^2$, then $\frac{d(y^{\frac{1}{2}})}{d(x^{\frac{1}{2}})}$ which of the following?

A. $1 - 2x$ B. $\frac{2x}{\sqrt{x - x^2}}$ C. $\frac{\sqrt{x}\,(1 - 2x)}{\sqrt{x - x^2}}$ D. $\frac{1 - 2x}{x^{\frac{1}{2}}}$

E. NOTA

11. Find $\frac{d^n y}{dx^n}$ if $y = \frac{1}{2 + x}$

A. $\frac{(-1)^n n!}{(2+x)^{n+1}}$ B. $\frac{(-1)^n n}{(2+x)^{n+1}}$ C. $\frac{n!}{(2+x)^{n+1}}$ D. $\frac{(-1)^n}{(2+x)^{n+1}}$ E. NOTA

12. The graph of $y = 5x^{2/3} - x^{5/3}$ has an inflection point (or points) at what value(s) of x ?

A. x= -1 only B. x= -1 and 0 C. x= 2 only D. x= 0 and 2 E. NOTA

13. $\sin(xy) = y^2$ find $\frac{dy}{dx}$

A. $\sqrt{1 - y^4}$ B. $\frac{2y}{\sqrt{1 - y^4}}$ C. $\frac{y}{2y\sqrt{1 - y^4} - x}$

D. $\frac{y\sqrt{1 - y^4}}{2y - x\sqrt{1 - y^4}}$ E. NOTA

14. $y = a\sin(ct) + b\cos(ct)$ where a,b, and c are constants. Then $\frac{d^2y}{dt^2}$ is:

A. $ac^2(\sin t + \cos t)$ B. $-c^2y$ C. $-ay$

D. $-y$ E. NOTA

15. $f(x) = kx^2 + c$. L_1 is tangent to the curve at $(x_0, f(x_0))$ and L_2 is tangent to the curve at $(-x_0, f(-x_0))$. L_1 and L_2 are perpendicular to each other. What are the coordinates of their point of intersection? (k and c are constants and L_1 and L_2 are lines.)

A. $(0, c - \frac{1}{4k})$ B. $(0, \frac{1}{4k})$ C. $(0, -c)$

D. $(0, c + \frac{1}{4k})$ E. NOTA

16. $x^{1/2} + y^{1/2} = 7$ Find the sum of the x and y intercepts of the tangent line to this curve at the point (9,16).

A. $-\frac{4}{3}$ B. 7 C. 25 D. 49 E. NOTA

17. The tangent line to the curve $y = x^4 + 3$ at the point (a,b) passes through the origin. Find the value of a + b, if the point (a,b) is in the second quadrant.

A. 0 B. 3 C. 5 D. 7 E. NOTA

18. Approximate the value of $6x^4 - 4x^3 + 5x^2 - 6$ when x = .97 using differentials.

A. .34 B. .36 C. 1.34 D. 1.36 E. NOTA

19. Find $\frac{dy}{dx}$ if $\sec^2 y + \csc^2 x = 3$

A. $\dfrac{-\csc^2 x \cot x}{\sec^2 y \tan y}$ B. $\cot^3(\frac{y}{x}) \tan(\frac{x}{y})$ C. $\dfrac{\cos^3 y \cos x}{\sin^3 x \sin y}$

D. $\dfrac{-\tan^2 y}{\cot^2 x}$ E. NOTA

20. Evaluate: $\int_0^3 x^2\sqrt{1+x}\,dx$

A. $\frac{10}{216}$ B. $\frac{52}{5}$ C. $\frac{63}{35}$ D. $\frac{1696}{105}$ E. NOTA

21. Evaluate: $\int_1^3 \frac{1}{y(1+y^2)}\,dy$

A. $\frac{1}{2}\ln\frac{2}{15}$ B. $\frac{1}{2}\ln\frac{9}{5}$ C. $\frac{1}{2}\ln\frac{15}{2}$ D. $\frac{1}{2}\ln 2$ E. NOTA

22. $4\int x^2e^{2x}dx = ax^2e^{2x} + bxe^{2x} + ce^{2x} + d$
What is the value of b + c ?

A. -1 B. 0 C. 1 D. 2 E. NOTA

23. Find the area between the graphs of $2y^2 = x + 4$ and $x = y^2$.

A. $\frac{16}{3}$ B. $\frac{32}{3}$ C. $\frac{64}{3}$ D. $\frac{72}{3}$ E. NOTA

24. $x + y = 20$ and $P = x^3y^7$ Find the value of y which will make P a maximum.

A. 6 B. 10 C. 15 D. 19 E. NOTA

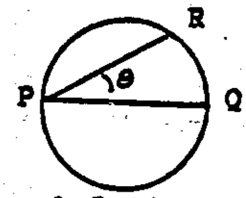
25. A stone is dropped from the top of a 144 ft. tall building. One second later, a second stone is thrown from the top of the same building. With what velocity must it be thrown so that the two stones will strike the ground at the same time? (use $g = -32$ ft/sec^2) Give your answer in ft/sec .

A. -40 B. -44 C. -58 D. -80 E. NOTA

26. A wall 8 ft. high is 3.375 ft. from a house. What is the length of the shortest ladder which will reach from the ground, over the wall and touch the house? Give your answer in feet.

A. 9.375 B. 15.625 C. 15.875 D. 18.5 E. NOTA

27. In the figure on the right, a man is at point P on a circular lake and wishes to get to point Q, directly across the lake from him. He decides to row to point R and then walk along the shore from R to Q. He can row 1.5 miles per hour and walk 3 miles per hour. What angle (in degrees) should θ be in order to minimize the time required to get from P to Q?



A. 0 B. 30 C. 45 D. 60 E. NOTA

28. Find the volume generated when the graph of $y = \dfrac{x^{3/2}}{\sqrt{x^2 + 4}}$ between $x = 0$ and $x = 4$ is rotated about the x-axis .

A. $2\pi(4 - \ln 5)$ B. $\pi(16 - \ln 5)$ C. $2\pi \ln 5$

D. 8π E. NOTA

29. The position of a particle at time t is given by the equations:

$$x = \tfrac{1}{2}t^2 \qquad \text{and} \qquad y = \tfrac{1}{9}(6t + 9)^{3/2}$$

How far does the particle travel from $t = 0$ to $t = 4$?

A. 3 B. 7.65 C. 20 D. 24 E. NOTA

30. Use the trapezoid rule with $n = 4$ to find the area under the curve $y = 2x^2 - 3x + 6$ with the boundaries being: x-axis and $x=0$ and $x=4$. The area you get is:

A. $\frac{121}{3}$ B. $\frac{128}{3}$ C. 44 D. $\frac{138}{3}$ E. NOTA