Middleton 3/5/05

March Regional Algebra II Team Test

Question # 1

Given: $A=\begin{bmatrix}1 & 5\\ 4 & -5\end{bmatrix}+\begin{bmatrix}-4 & 0\\ 3 & -2\end{bmatrix}$

$B=\begin{bmatrix}-3 & 4\\ 3 & -5\end{bmatrix}\begin{bmatrix}-5 & 2\\ 1 & -2\end{bmatrix}$

$C=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}^{-1}$

$D=\det\left(\begin{bmatrix}-5 & 4\\ -2 & 1\end{bmatrix}\right)$

Find: $\det(A+B+DC)$

Question # 2

Let
A = the sum of the first 10 prime numbers
B = the sum of the first 4 triangular numbers
C = the product of the first 6 Fibonacci numbers
D = the sum of the first 8 positive odd integers
Find the simplified value of the expression

$$\frac{B+C}{A-D}$$

Question # 3

At the FAMAT state convention a relay team consists of a geometry seat, an algebra II seat, a precalculus seat, and a calculus seat. An algebra II student can be in any seat. A precalculus student can only be in the precalculus or the calculus seat. A calculus student can only be in the calculus seat. If a school brings 4 algebra II, 4 precalculus, and 4 calculus students to the convention, how many different relay teams can they form? (Note: a team is different if it does not have the same person in each seat.)

Question # 4

Let A, B, and C be, respectively, the solution sets for equations I, II, and III below.

If $X = A \cup B \cup C$, find the product of all elements in set X.

I: $|3x+2|=4x+5$

II: $\frac{x}{x-3}+\frac{1}{x+1}=\frac{12}{x^2-2x-3}$

III. $x-\sqrt{10-3x}=2$

Question # 5

Given: $f(x)=|x|$ $\quad g(x)=\log_{10}x$

$h(x)=x^2-4$

A = domain of $f(g(h(x)))$

B = range of $f(g(h(x)))$

Find: $A \cap B$

Question # 6

Given:

A = constant term in $\left(x^2+\frac{2}{x}\right)^9$

B = sum of coefficients in $(x+y)^6$

Find: $\frac{A}{B}$

Question # 7

At the awards ceremony today, you and exactly 3 other students from your school are among 15 students that get called up to receive Algebra II individual awards.

Given:
A = number of ways awards can be given out to the 15 individuals
B = number of ways awards can be given out if you receive 1st place
C = number of ways awards can be given out if the top 4 individuals are from your school
D = number of ways awards can be given out if the top 2 individuals are from your school

Find: $\frac{AD}{BC}$

Question # 8

Find the sum of the integers in the solution of the system:

$x^2-4x<-2$

$\log_{10}\left(\frac{x-2}{2}\right)\le 0$.

Question # 9

What is the area in the x-y plane of the solution to the system:

$-3x+11y<30$

$|x-5|-1<y$

Middleton 3/5/05

March Regional Algebra II Team Test

Question # 10

Given: $f(x) = (\log_2 x)^3$ $\quad g(x) = 4^{\frac{x-3}{2}}$

$A = f(32)$ $\quad B = f^{-1}(27)$

$C = g(-4)$ $\quad D = g^{-1}(8)$

Find: $\dfrac{AD}{BC}$

Question # 11

The nth term of a real sequence is denoted by a_n.

Given: $a_1 = 2$ $\quad a_4 = 16$

$A = a_{10}$ if the sequence is arithmetic

$B = a_{10}$ if the sequence is geometric

$C = \sum_{n=1}^{10} a_n$ if the sequence is arithmetic

$D = \sum_{n=1}^{10} a_n$ if the sequence is geometric

Find: $A + B + C + D$

Question # 12

Find:

$$\sum_{n=1}^{790} i^n$$

Question # 13

Given: $f(x) = \dfrac{1 - \dfrac{1}{1+x}}{\dfrac{1}{1+x}}$

If $A = \sum_{x=1}^{40} f(x)$ and B = the x-intercept of $f(x)$,

Find the value of $A(B+3)$

Question # 14

Find: $\begin{vmatrix} 1 & i & i^2 & i^3 \\ i^4 & i^5 & i^6 & i^7 \\ i^8 & i^9 & i^{10} & i^{11} \\ i^{12} & i^{13} & i^{14} & i^{15} \end{vmatrix}$

Question # 15

Planets follow elliptical paths where the center of the star they orbit is at a focus of the ellipse.

A = minimum distance from the star to the planet if the planet's orbit has a semimajor axis of 10

B = maximum distance from the star to the planet in A

C = x-coordinate of the star if the paths of two planets that orbit the same star in the same plane are described by $9x^2 + 25y^2 - 18x - 200y + 184 = 0$ and $25x^2 + 9y^2 - 250x + 400 = 0$

Find: $A + B + C$