MIDDLETON

MARCH 5, 2005

ALGEBRA 2

TIGERS

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F.A.M.A.T
REGIONAL
COMPETITION

TIGERS

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March Regional Algebra II Individual Test

For all questions, E. NOTA means none of the above answers is correct. Let $i = \sqrt{-1}$.

1. If $f(x) = 4x^2 - 8x + 1$, what is $f(0)$?

   A) -8 B) -1 C) 1 D) 4 E) NOTA

2. Find: $(i^{27}) \cdot (i^8)$

   A) $-i$ B) -1 C) 1 D) i E) NOTA

3. Solve: $x^2 - 4x - 5 > 0$

   A) $(-1,5)$ B) $[-1,5]$ C) $(-\infty,-5) \cup (1,\infty)$
   D) $(-\infty,-5] \cup [1,\infty)$ E) NOTA

4. Which of the following could not be the possible number of positive real solutions of the following equation where a, b, c, d, e, and f are positive numbers?

   $$x^7 - ax^6 + bx^5 - cx^4 + dx^3 + ex - f = 0$$

   A) 0 B) 1 C) 3 D) 5 E) NOTA

5. Steven is trying to make a gallon of Gatorade. You are supposed to mix 4 scoops of Gatorade powder with water to make a gallon of Gatorade. After mixing 4 scoops and half a gallon of water, Florian drinks 1 pint of the mixture. Then, Steven adds more water until he has 1 gallon. How many more scoops of powder should Steven add to properly make a gallon of Gatorade? (There are 8 pints in a gallon. Assume the volume of Gatorade powder is negligible.)

   A) 0.5 B) 0.75 C) 1 D) 1.25 E) NOTA

6. Which of the following matrices does not have a multiplicative inverse?

   A) $\begin{bmatrix} -4 & -2 \\ -2 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$ D) $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ E) NOTA

7. How many of the following are true for all positive numbers x, y, and z where x, y, and z are not equal to 1?

   I. $\log_x x = 1$

   II. $\log_x (yz) = \log_x y + \log_x z$

   III. $x = y^{\frac{1}{\log_x y}}$

   IV. $(\log_x y)^z = z \cdot \log_x y$

   A) 1 B) 2 C) 3 D) 4 E) NOTA

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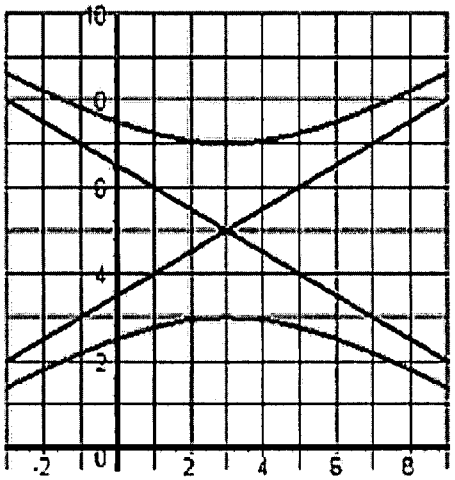
8. Given that a, b, and c are integers where a is odd and b is a multiple of 3, how many rational numbers could be a root of the following equation?

$$x^4 + \frac{ax^3}{2} + \frac{bx^2}{3} + cx + 5 = 0$$

A) 4 B) 12 C) 16 D) 36 E) NOTA

9. Refer to the graph below showing a hyperbola and its asymptotes. Express the equation of the hyperbola graphed below in the form $x^2 + ay^2 + bx + cy + d = 0$, where a,b,c,d are integers. Find $a+b+c+d$.



A) -77 B) -68 C) -45 D) -35 E) NOTA

10. Find the vertex of the parabola $4x^2 + 16x - 16y = 32$

A) $(-3,-2)$ B) $(-2,-3)$ C) $(2,3)$ D) $(3,2)$ E) NOTA

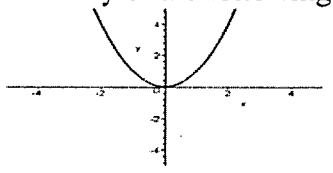
11. Given that a_i represents the i^{th} term of a geometric sequence, $a_1 = 1$, and $a_5 = \frac{1}{16}$, and that the common ratio is real and positive, what is the value of $\sum_{i=1}^{\infty} a_i$?

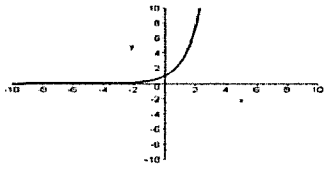
A) 2 B) $\frac{8}{3}$ C) $\frac{18}{5}$ D) $\frac{64}{15}$ E) NOTA

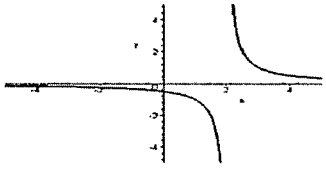
12. Let $f(x) = \frac{x^3 - 2x^2 + qx + r}{(x-3)(x+1)}$ where q and r are real numbers. Given that f(x) has no vertical asymptotes, find f(1).

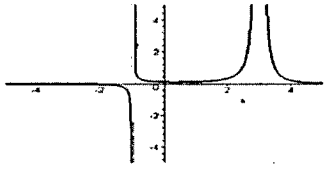
A) -1 B) 0 C) 1 D) 3 E) NOTA

13. How many of the following functions have an inverse that is a function?

I. 

II. 

III. 

IV. 

A) 0 B) 1 C) 2 D) 3 E) NOTA

14. The electric force between two objects is directly proportional to the product of the electric charges of the objects and inversely proportional to the square of the distance between the objects. If the charge on both objects is doubled and the distance between the objects is tripled, what is the ratio of the new force to the original force?

A) $\frac{2}{9}$ B) $\frac{4}{9}$ C) $\frac{2}{3}$ D) $\frac{4}{3}$ E) NOTA

15. What is the coefficient of x^{16} in the expansion of the following expression?

$$(x^2+2x)^{10}$$

A) 210 B) 960 C) 6,720 D) 8,064 E) NOTA

16. A dartboard is outlined by the graph of $9x^2+4y^2=36$. If you randomly hit the dartboard, what is the probability you are inside the bull's eye bounded by $x^2+y^2=1$?

A) $\frac{1}{6}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{2}{3}$ E) NOTA

17. Find:

$$\frac{(10+4i)(3-6i)}{3+9i}$$

A) $-\frac{7}{5}-\frac{19}{5}i$ B) $4+6i$ C) $-3-7i$ D) $18-\frac{16}{3}i$ E) NOTA

18. Find y.

$$\begin{bmatrix} -1 & -2 & 3 \\ 4 & 3 & 0 \\ 2 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$$

A) $-\frac{13}{9}$ B) $-\frac{3}{4}$ C) $\frac{9}{16}$ D) $\frac{11}{16}$ E) NOTA

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19. A school brings 6 theta, 9 alpha, and 16 calculus students to the state convention. If 4 students from the school are randomly selected to work on the poster, what is the probability that at least two divisions are represented? (Round to three significant digits.)

A) 0.0623 B) 0.0695 C) 0.930 D) 0.938 E) NOTA

20. Simplify:

$$\log_4 5 + \log_2 25$$

A) $\frac{2}{5}\log_5 2$ B) $\frac{2}{5}\log_2 5$ C) $\frac{5}{2}\log_5 2$ D) $\frac{5}{2}\log_2 5$ E) NOTA

21. Let $f(x) = 2x^3 + ax^2 + bx + c$ where a, b, and c are real numbers. The sum and the product of the roots of f(x) are 6 and -5, respectively. Given that 2 is a root of f(x), what is the value of b?

A) -22 B) -17.5 C) 11 D) 21 E) NOTA

22. Find the area of the triangle with vertices at the origin and the foci of $25x^2 + 9y^2 - 250x + 36y + 436 = 0$?

A) 4 B) $\sqrt{34}$ C) 20 D) $5\sqrt{34}$ E) NOTA

23. A bag contains 6 identical red marbles, 4 identical blue marbles, and 8 identical purple marbles. In how many different distinguishable orders can you take all of the marbles out of the bag by removing one at a time?

A) 9,189,180 B) 56,808,540 C) 133,899,480
D) 387,420,489 E) NOTA

24. Solve:

$$|2x - 5| \le \frac{1}{2}$$

A) $\left(\frac{9}{4}, \frac{11}{4}\right)$ B) $\left[\frac{9}{4}, \frac{11}{4}\right]$ C) $\left(-\infty, \frac{9}{4}\right) \cup \left(\frac{11}{4}, \infty\right)$
D) $\left(-\infty, \frac{9}{4}\right] \cup \left[\frac{11}{4}, \infty\right)$ E) NOTA

25. Given: $f(x) = x^3$
$g(x) = x + 1$
Find: $(f \circ g)(2)$

A) -2 B) 3 C) 9 D) 27 E) NOTA

26. What is the 5^{th} term of an arithmetic sequence with a common difference of 5 and first term of 2?

A) 13 B) 15 C) 22 D) 27 E) NOTA

27. Given that $f(x+1)f(x) = x$ for all positive integers x. What is the ten thousandths digit of $f(2001)f(2004)$?

A) 1 B) 5 C) 7 D) 9 E) NOTA

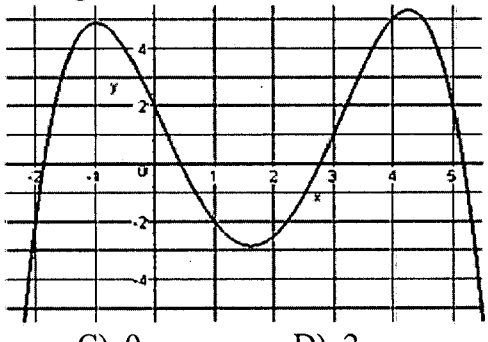
28. What is the probability of rolling a prime number on a fair six-sided die?

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{5}{6}$ D) 1 E) NOTA

29. What is the slope between $(-2,6)$ and $(c,3)$?

A) $\frac{-c-2}{3}$ B) $\frac{-3}{c+2}$ C) $\frac{3}{c+2}$ D) $\frac{c+2}{3}$ E) NOTA

30. What is the sum of the coefficients excluding the constant term of the polynomial graphed below? (The polynomial has integral values at $x = -2, 0, 1, 3, 4,$ and 5.)



A) -4 B) -2 C) 0 D) 2 E) NOTA

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1. $f(0)=4*0^2-8*0+1=1$
   Answer: 1; C
2. $i^{27}*i^8=i^{27+8}=i^{35}=-i$
   Answer: $-i$; A
3. $x^2-x-5>0$
   $(x-5)(x+1)>0$ Roots: -1,5
   Parabola opens up so it's the outside intervals.
   Answer: $(-\infty,-1)\cup(5,\infty)$; E
4. There are 5 sign changes in f(x) and 0 sign changes in f(-x). Since complex numbers always come in pairs there must be an odd number of positive solutions according to Descartes' Rule of Signs.
   Answer: 0; A
5. He drinks 1 pint out of 4 pints, so he drinks $\frac{1}{4}$ of the Gatorade powder in the mixture. Therefore, he drinks $4*\frac{1}{4}=1$ scoop of Gatorade powder.
   Answer: 1; C
6. A matrix does not have a multiplicative inverse iff the determinant of the matrix is 0.
   Answer: $\begin{bmatrix}3 & 6\\ 2 & 4\end{bmatrix}$; D
7. I, II, and III
   Answer: 3; C
8. First multiply by 2 to get integers coefficient ($\frac{2b}{3}$ is an integer since c is a multiple of 3):
   $2x^4+ax^3+\frac{2b}{3}x^2+2cx+10=0$
   The rational root theorem says all possible rational roots are factors of the constant term divided by factors of the leading term. The possible roots are: $\pm10,\pm5,\pm2,\pm1,\pm\frac{5}{2},\pm\frac{1}{2}$
   Answer: 12; B
9. $-\left(\frac{x-3}{4}\right)^2+\left(\frac{y-5}{2}\right)^2=1$
   $-(x^2-6x+9)+4(y^2-10y+25)=16$
   $-x^2+4y^2+6x-40y-9+100=16$
   $x^2-4y^2-6x+40y-75=0$
   $-4-6+40-75=-45$
   Answer: -45; C
10. $4x^2+16x-16y=32$
    $x^2+4x-4y=8$
    $x^2+4x+4=4y+12$
    $(x+2)^2=4(y+3)$
    Answer: $(-2,-3)$, B
11. $r^4a_1=a_5$
    $r^4=\frac{1}{16}$
    $r=\frac{1}{2}$
    $\sum_{i=1}^{\infty}a_i=\frac{a_1}{1-r}=2$
    Answer: 2; A
12. 3 and -1 are roots of the numerator. We know that the sum of the roots of the numerator is 2, so the third root must be 0. Where defined f(x)=x, so f(1)=1.
    Answer: 1; C
13. II and III (Horizontal line test.)
    Answer: 2; C
14. $\frac{2*2}{3^2}=\frac{4}{9}$
    Answer: $\frac{4}{9}$; B
15. It is easiest to first factor out an x.
    $(x^2+2x)^{10}=x^{10}(x+2)^{10}$ Now the answer is what is the coefficient of x^6 in $(x+2)^{10}$. $\binom{10}{6}*2^4=3{,}360$
    Answer: 3,360; E

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16. $9x^2+4y^2=36$

$\left(\frac{x}{2}\right)^2+\left(\frac{y}{3}\right)^2=1$

Dartboard is ellipse with area $3*2*\pi=6\pi$. Bull's eye is circle with radius 1 and has area π. Probability of getting a bull's eye is $\frac{\pi}{6\pi}=\frac{1}{6}$

Answer: $\frac{1}{6}$; A

17. $\frac{(10+4i)(3-6i)}{3+9i}=\frac{(10+4i)(1-2i)}{1+3i}=$

$\frac{18-16i}{1+3i}*\frac{1-3i}{1-3i}=\frac{-30-70i}{10}=-3-7i$

Answer: $-3-7i$; C

18. Cramer's Rule

$$y=\frac{\begin{vmatrix}-1&3&3\\4&0&0\\2&1&-2\end{vmatrix}}{\begin{vmatrix}-1&-2&3\\4&3&0\\2&1&-2\end{vmatrix}}=\frac{12}{-16}=-\frac{3}{4}$$

Answer: $-\frac{3}{4}$; B

19. Answer is 1-probability of all 4 students coming from 1 division.

$$1-\frac{\binom{6}{4}+\binom{9}{4}+\binom{16}{4}}{\binom{6+9+16}{4}}\approx 0.938$$

Answer: 0.938; D

20. $\log_4 5+\log_2 25=\frac{1}{2}\log_2 5+2\log_2 5$

$\frac{5}{2}\log_2 5$

Answer: $\frac{5}{2}\log_2 5$; D

21. Sum of roots gives $-\frac{a}{2}=6$, $a=-12$.

Product of roots gives $-\frac{c}{2}=-5$, $c=10$. Since 2 is a root, $2*8-12*4+2b+10=0$ $b=11$

Answer: 11; C

22. $25x^2+9y^2-250x+36y+436=0$

$25(x^2-10x+25)+9(y^2+4y+4)=225$

$\left(\frac{x-5}{3}\right)^2+\left(\frac{y+2}{5}\right)^2=1$

Foci are at $(5,-2\pm 4)$. Area of triangle is $\frac{5*8}{2}=20$

Answer: 20; C

23. $\binom{6+4+8}{6}\binom{4+8}{4}\binom{8}{8}=9{,}189{,}180$

Answer: 9,189,180; A

24. $|2x-5|\le\frac{1}{2}$ $\qquad 2\left|x-\frac{5}{2}\right|\le\frac{3}{2}$

$\left|x-\frac{5}{2}\right|\le\frac{3}{4}$

The solution is a closed interval centered at $\frac{5}{2}$ with radius $\frac{1}{4}$.

$\left[\frac{5}{2}-\frac{1}{4},\frac{5}{2}+\frac{1}{4}\right]=\left[\frac{9}{4},\frac{11}{4}\right]$

Answer: $\left[\frac{9}{4},\frac{11}{4}\right]$; B

25. $g(2)=2+1=3$

$f(3)=3^3=27$

Answer: 27; D

26. $a_n=a_1+(n-1)d$

$a_5=2+(5-1)5$

$a_5=22$

Answer: 22; C

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27. $f(x) = \dfrac{x}{f(x+1)}$

$f(2002) = \dfrac{2002}{f(2003)} = \dfrac{2002 * f(2004)}{2003}$

$f(2001) = \dfrac{2001}{f(2002)} = \dfrac{2001 * 2003}{2002 * f(2004)}$

$f(2001)f(2004) = \dfrac{2001 * 2003}{2002} \approx$

2001.9995

Answer: 5; B

28. There are only 3 primes less than or including 6: 2,3, and 5. The probability is $\dfrac{3}{6} = \dfrac{1}{2}$

Answer: $\dfrac{1}{2}$; A

29. $m = \dfrac{\Delta y}{\Delta x} = \dfrac{6-3}{-2-c} = \dfrac{-3}{c+2}$

Answer: $\dfrac{-3}{c+2}$; B

30. The sum of all of the coefficients (including the constant term) of a polynomial is f(1). The constant term is given by f(0).

$f(1) - f(0) = -2 - 2 = -4$

Answer: -4; A

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March Regional Algebra II Team Test

Question # 1

Given: $A=\begin{bmatrix}1 & 5\\ 4 & -5\end{bmatrix}+\begin{bmatrix}-4 & 0\\ 3 & -2\end{bmatrix}$

$B=\begin{bmatrix}-3 & 4\\ 3 & -5\end{bmatrix}\begin{bmatrix}-5 & 2\\ 1 & -2\end{bmatrix}$

$C=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}^{-1}$

$D=\det\left(\begin{bmatrix}-5 & 4\\ -2 & 1\end{bmatrix}\right)$

Find: $\det(A+B+DC)$

Question # 2

Let
A = the sum of the first 10 prime numbers
B = the sum of the first 4 triangular numbers
C = the product of the first 6 Fibonacci numbers
D = the sum of the first 8 positive odd integers
Find the simplified value of the expression

$$\frac{B+C}{A-D}$$

Question # 3

At the FAMAT state convention a relay team consists of a geometry seat, an algebra II seat, a precalculus seat, and a calculus seat. An algebra II student can be in any seat. A precalculus student can only be in the precalculus or the calculus seat. A calculus student can only be in the calculus seat. If a school brings 4 algebra II, 4 precalculus, and 4 calculus students to the convention, how many different relay teams can they form? (Note: a team is different if it does not have the same person in each seat.)

Question # 4

Let A, B, and C be, respectively, the solution sets for equations I, II, and III below.

If $X = A \cup B \cup C$, find the product of all elements in set X.

I: $|3x+2|=4x+5$

II: $\frac{x}{x-3}+\frac{1}{x+1}=\frac{12}{x^2-2x-3}$

III. $x-\sqrt{10-3x}=2$

Question # 5

Given: $f(x)=|x|$ $\quad g(x)=\log_{10}x$

$h(x)=x^2-4$

A = domain of $f(g(h(x)))$

B = range of $f(g(h(x)))$

Find: $A \cap B$

Question # 6

Given:

A = constant term in $\left(x^2+\frac{2}{x}\right)^9$

B = sum of coefficients in $(x+y)^6$

Find: $\frac{A}{B}$

Question # 7

At the awards ceremony today, you and exactly 3 other students from your school are among 15 students that get called up to receive Algebra II individual awards.

Given:
A = number of ways awards can be given out to the 15 individuals
B = number of ways awards can be given out if you receive 1st place
C = number of ways awards can be given out if the top 4 individuals are from your school
D = number of ways awards can be given out if the top 2 individuals are from your school

Find: $\frac{AD}{BC}$

Question # 8

Find the sum of the integers in the solution of the system:

$x^2-4x<-2$

$\log_{10}\left(\frac{x-2}{2}\right)\le 0$.

Question # 9

What is the area in the x-y plane of the solution to the system:

$-3x+11y<30$

$|x-5|-1<y$

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Question # 10

Given: $f(x) = (\log_2 x)^3$ $\quad g(x) = 4^{\frac{x-3}{2}}$

$A = f(32)$ $\quad B = f^{-1}(27)$

$C = g(-4)$ $\quad D = g^{-1}(8)$

Find: $\dfrac{AD}{BC}$

Question # 11

The nth term of a real sequence is denoted by a_n.

Given: $a_1 = 2$ $\quad a_4 = 16$

$A = a_{10}$ if the sequence is arithmetic

$B = a_{10}$ if the sequence is geometric

$C = \sum_{n=1}^{10} a_n$ if the sequence is arithmetic

$D = \sum_{n=1}^{10} a_n$ if the sequence is geometric

Find: $A + B + C + D$

Question # 12

Find:

$$\sum_{n=1}^{790} i^n$$

Question # 13

Given: $f(x) = \dfrac{1 - \dfrac{1}{1+x}}{\dfrac{1}{1+x}}$

If $A = \sum_{x=1}^{40} f(x)$ and B = the x-intercept of $f(x)$,

Find the value of $A(B+3)$

Question # 14

Find: $\begin{vmatrix} 1 & i & i^2 & i^3 \\ i^4 & i^5 & i^6 & i^7 \\ i^8 & i^9 & i^{10} & i^{11} \\ i^{12} & i^{13} & i^{14} & i^{15} \end{vmatrix}$

Question # 15

Planets follow elliptical paths where the center of the star they orbit is at a focus of the ellipse.

A = minimum distance from the star to the planet if the planet's orbit has a semimajor axis of 10

B = maximum distance from the star to the planet in A

C = x-coordinate of the star if the paths of two planets that orbit the same star in the same plane are described by $9x^2 + 25y^2 - 18x - 200y + 184 = 0$ and $25x^2 + 9y^2 - 250x + 400 = 0$

Find: $A + B + C$

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March Regional Algebra II Team Solutions

1. $A=\begin{bmatrix}-3 & 5\\ 7 & -7\end{bmatrix}$

$B=\begin{bmatrix}19 & -14\\ -20 & 16\end{bmatrix}$

$C=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$

$D=-5*1+2*4=3$

$\det(A+B+CD)=\det\left(\begin{bmatrix}16 & -6\\ -10 & 9\end{bmatrix}\right)=16*9+10*-6=84$

Answer: **84**

2. A = 2+3+5+7+11+13+17+19+23+29 = 129
B = 1+3+6+10 = 20
C = 1x1x2x3x5x8 = 240
D = 8^2 = 64
So (B+C)/(A-D) = 260/65 = 4
Answer : **4**

3. One of the 4 algebra II students has to sit in the geometry seat and one of the 3 remaining algebra II students has to sit in the algebra II seat. The 4 precalculus students or the remaining 2 algebra II students may sit in the precalculus seat. Finally, there are together 5 precalculus and algebra II students and 4 calculus students that may be in the calculus seat. $4*3*6*9=648$
Answer: **648**

4 I.: Solve 3x+2 = 4x +5 and 3x+2 = -4x-5; the only valid solution is x = -1
II: Mult. by LCD gives:
$x^2+x+x-3=12\Rightarrow x^2+2x-15=0\Rightarrow$
$x=-5,3;$ *only* -5 *is a valid root.*

III. $(x-2)^2=10-3x$ gives x = 3, -2, and only 3 is a valid root.
$A\cup B\cup C=\{-1,-5,3\}$ The product of these elements is **15**
Answer: 15

5. Domain of $\log_{10}x$ is $x>0$ so the domain of $f(g(h(x)))$ is $x^2-4>0$
$A=(-\infty,-2)\cup(2,\infty)$
Since the range of $|x|$ is $[0,\infty)$
$B=[0,\infty)$
$A\cap B=(2,\infty)$
Answer: $(2,\infty)$ or $x>2$

6. $A=\binom{9}{3}\left(x^2\right)^3\left(\frac{2}{x}\right)^6=\binom{9}{3}*2^6=84*2^6$

$B=(1+1)^6=2^6$

$\frac{A}{B}=\frac{84*2^6}{2^6}=84$

Answer: 84

7. $A=15!\quad B=14!\quad C=4!*11!$

$D=\binom{4}{2}*2*13!=12*13!$

$\frac{AD}{BC}=\frac{15!*12*13!}{14!*4!*11!}=1{,}170$

Answer: **1,170**

8. $x^2-4x+2<0$

$A=\left(2-\sqrt{2},2+\sqrt{2}\right)$

$\log_{10}\left(\frac{x-2}{2}\right)\le 0$

$\frac{x-2}{2}\le 1\qquad x\le 4$

In order to be in the domain $\frac{x-2}{2}>0$ or $2<x$. $B=(2,4]$

$A\cap B=\left(2,2+\sqrt{2}\right)$

Answer: **3**

9. The area is a right triangle bounded by the intersection of $-3x+11y=30$ and $|x-5|-1=y$ and the minimum of the absolute value equation, (5,-1). Substitution gives the intersection:

$\underline{x-5>0}$

$-3x+11(x-5-1)=30$

$-3x+11x-66=30$

$x=12,\ y=6$

$\underline{x-5<0}$

$-3x+11(-x+5-1)=30$

$-3x-11x+44=30$

$x=1,\ y=3$

The area of the triangle is half the product of the legs:

$.5*\sqrt{(5-12)^2+(-1-6)^2}*\sqrt{(5-1)^2+(-1-3)^2}=$

$.5*7\sqrt{2}*4\sqrt{2}=28$

Answer: **28**

Middleton 3/5/05

March Regional Algebra II Team Solutions

10. $A = (\log_2 32)^3 = 5^3 = 125$

$f^{-1}(x) = 2^{\sqrt[3]{x}}$

$B = f^{-1}(27) = 2^{\sqrt[3]{27}} = 8$

$C = 4^{\frac{-4-3}{2}} = 2^{-7} = \frac{1}{128}$

$g^{-1}(x) = 2 * \log_4 x + 3 = \log_2 x + 3$

$D = \log_2 8 + 3 = 3 + 3 = 6$

$\frac{AD}{BC} = \frac{125 * 6 * 128}{8} = 12{,}000$

Answer: **12,000**

11. $a_4 = a_1 + 3d ; 16 = 2 + 3d ; d = \frac{14}{3}$

$A = a_{10} = a_1 + 9d = 2 + 9 * \frac{14}{3} = 44$

$a_4 = a_1 r^{n-1} \qquad 16 = 2r^3 \qquad r = 2$

$B = a_{10} = a_1 r^{10-1} = 2 * 2^9 = 1{,}024$

$C = 10 * \frac{2 + 44}{2} = 230$

$D = a_1 \frac{1 - r^{10}}{1 - r} = 2 * \frac{1 - 2^{10}}{1 - 2} = 2{,}046$

$A + B + C + D = 3{,}344$

Answer: **3,344**

12. $\sum_{n=1}^{4} i^n = i - 1 - i + 1 = 0$

Thus, we can ignore groups of 4 consecutive numbers since they will add to 0. Since $790 \equiv 2$ in modulus 4, we have

$\sum_{n=1}^{790} i^n = \sum_{n=789}^{790} i^n = i + i^2 = i - 1$

Answer: $i - 1$

13. Simplifying, we get that $f(x) = x$. So,

$A = \sum_{x=1}^{40} x = 20 \cdot 41 = 820$ and $B = 0$

So,

$A(B+3) = 820(0+3) =$ **2,460**

14. Reduce the powers of i that are greater than 4:

$$\begin{vmatrix} 1 & i & i^2 & i^3 \\ 1 & i & i^2 & i^3 \\ 1 & i & i^2 & i^3 \\ 1 & i & i^2 & i^3 \end{vmatrix}$$

Since there are repeated rows in this matrix, the determinant is 0. (Subtract one row from another and expand along the resulting 0 row.)

Answer: **0**

15. A = semimajor axis - focal distance

B = semimajor axis + focal distance

A + B = 2*semimajor axis=20

x-coordinate of star is same as x-coordinate of the center of the ellipse with horizontal major axis, which is the 2nd ellipse

C = 5

A + B + C = 20 + 5 = 25

Answer: **25**