

MU ALPHA THETA REGIONAL COMPETITION  
MARCH 20, 1993 - Part

4

ALGEBRA II TEAM TEST

1) Simplify:  $\left(\frac{3^2 - 3^0}{9^{\frac{3}{2}}}\right)^{-\frac{1}{3}}$

2) Find the smallest positive integer  $n$  for which  $\frac{7!}{n}$  is not an integer.

3) A = The coefficient of the seventh term of  $(x - 2y)^6$

B = The coefficient of the twelfth term of  $(2 + x)^{11}$

C = The coefficient of the first term of  $(2x + 3y)^4$

Find  $\frac{AB}{C}$

4) Find  $n$  if  $(123_{\text{base } n})(234_{\text{base } n}) = 40442_{\text{base } n}$   
(Assume  $n$  must be positive)

5) Find the real number  $A$  in the following equation:

$$\frac{A}{x-4} + \frac{B}{x+3} = \frac{3x+2}{(x-4)(x+3)}$$

6)  $A = i^{77} + i^{12} + i^{186} + 2i^{85}$

B = The maximum number of positive real solutions of  $2x^4 - x^3 - x + 3 = 0$

C = The value of  $x$  in the equation  $x + 2\sqrt{x} - 3 = 0$

D = The value of  $k$  for which there will be only one solution of  $kx^2 - 4x = 2$

Find  $AC + B + D$

7) A = The radius of the circle  $x^2 + y^2 - 6x - 8y + 24 = 0$

B = The vertex of the parabola  $x^2 - 8x - y + 19 = 0$

C = The center of the ellipse  $x^2 + 4y^2 + 6x - 7 = 0$

D = The distance from point B to point C

Find the value of AD

8) If the graphs of  $f(x) = ax + b$  and  $g(x) = bx - a$  are perpendicular lines, express  $f(0) \cdot g(0)$  as an integer.

9) If  $f\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$ , find  $f\left(2\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)$ .

10) The sum of the digits of a three-digit number is 10. The sum of the tens digit and the units digit equals the hundreds digit. The number represented if the hundreds digit is omitted is one less than three times the hundreds digit. Find the three-digit number.

11) A = The eighth Fibonacci number

B = The sum of the first 100 odd integers

C =  $\lceil \sqrt{(3!)!} \rceil$  where  $[x]$  denotes the greatest integer function

D =  $[x + 1] - [x]$  where  $[x]$  denotes the greatest integer function

Find  $\frac{AC}{BD}$  expressed in decimal form

12) A:  $423_5 + 303_5 = \underline{\quad}_5$

B:  $816_{10} = \underline{\quad}_5$

C:  $101110_2 = \underline{\quad}_5$

Find  $(A + B + C)_5$

13) Let  $P(x) = (((((2)x + 4)x - 13)x - 5)x - 2)$  and  $Q(x) = 2x^4 + 4x^3 - 13x^2 - 5x - 3$ .

Find  $P(2) + Q(2)$ .

14) Solve for x:  $(x - 4)^{-2} (x + 2)^3 (x - 3)^2 = 0$

15) Write a linear function (f) such that  $f(4) = 16$  and  $f(-2) = 10$ .