

March Regional Algebra II Individual Test

For all questions, E. NOTA means none of the above answers is correct. Let $i = \sqrt{-1}$.

1. If $f(x) = 4x^2 - 8x + 1$, what is $f(0)$?
 A) -8 B) -1 C) 1 D) 4 E) NOTA

2. Find: $(i^{27}) \cdot (i^8)$
 A) $-i$ B) -1 C) 1 D) i E) NOTA

3. Solve: $x^2 - 4x - 5 > 0$
 A) $(-1, 5)$ B) $[-1, 5]$ C) $(-\infty, -5) \cup (1, \infty)$
 D) $(-\infty, -5] \cup [1, \infty)$ E) NOTA

4. Which of the following could not be the possible number of positive real solutions of the following equation where $a, b, c, d, e,$ and f are positive numbers?

$$x^7 - ax^6 + bx^5 - cx^4 + dx^3 + ex - f = 0$$

 A) 0 B) 1 C) 3 D) 5 E) NOTA

5. Steven is trying to make a gallon of Gatorade. You are supposed to mix 4 scoops of Gatorade powder with water to make a gallon of Gatorade. After mixing 4 scoops and half a gallon of water, Florian drinks 1 pint of the mixture. Then, Steven adds more water until he has 1 gallon. How many more scoops of powder should Steven add to properly make a gallon of Gatorade? (There are 8 pints in a gallon. Assume the volume of Gatorade powder is negligible.)
 A) 0.5 B) 0.75 C) 1 D) 1.25 E) NOTA

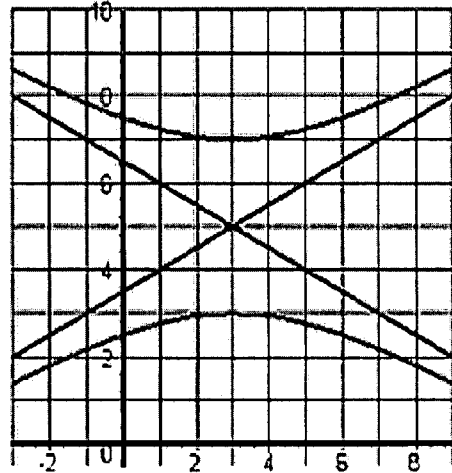
6. Which of the following matrices does not have a multiplicative inverse?
 A) $\begin{bmatrix} -4 & -2 \\ -2 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$ D) $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ E) NOTA

7. How many of the following are true for all positive numbers $x, y,$ and z where $x, y,$ and z are not equal to 1?
 I. $\log_x x = 1$
 II. $\log_x (yz) = \log_x y + \log_x z$
 III. $x = y^{\frac{1}{\log_x y}}$
 IV. $(\log_x y)^z = z \cdot \log_x y$
 A) 1 B) 2 C) 3 D) 4 E) NOTA

8. Given that a , b , and c are integers where a is odd and b is a multiple of 3, how many rational numbers could be a root of the following equation?

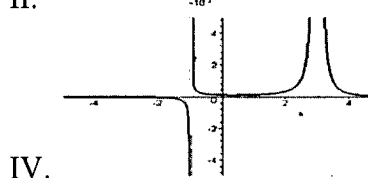
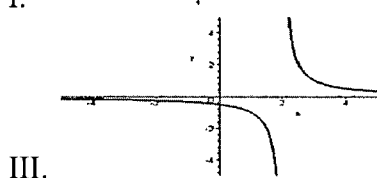
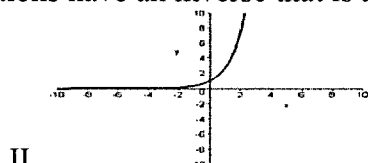
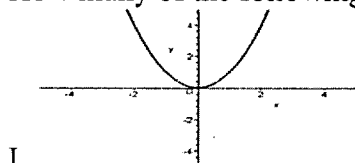
$$x^4 + \frac{ax^3}{2} + \frac{bx^2}{3} + cx + 5 = 0$$

- A) 4 B) 12 C) 16 D) 36 E) NOTA
9. Refer to the graph below showing a hyperbola and its asymptotes. Express the equation of the hyperbola graphed below in the form $x^2 + ay^2 + bx + cy + d = 0$, where a, b, c, d are integers. Find $a + b + c + d$.



- A) -77 B) -68 C) -45 D) -35 E) NOTA
10. Find the vertex of the parabola $4x^2 + 16x - 16y = 32$
- A) (-3, -2) B) (-2, -3) C) (2, 3) D) (3, 2) E) NOTA
11. Given that a_i represents the i^{th} term of a geometric sequence, $a_1 = 1$, and $a_5 = \frac{1}{16}$, and that the common ratio is real and positive, what is the value of $\sum_{i=1}^{\infty} a_i$?
- A) 2 B) $\frac{8}{3}$ C) $\frac{18}{5}$ D) $\frac{64}{15}$ E) NOTA
12. Let $f(x) = \frac{x^3 - 2x^2 + qx + r}{(x-3)(x+1)}$ where q and r are real numbers. Given that $f(x)$ has no vertical asymptotes, find $f(1)$.
- A) -1 B) 0 C) 1 D) 3 E) NOTA

13. How many of the following functions have an inverse that is a function?



- A) 0 B) 1 C) 2 D) 3 E) NOTA

14. The electric force between two objects is directly proportional to the product of the electric charges of the objects and inversely proportional to the square of the distance between the objects. If the charge on both objects is doubled and the distance between the objects is tripled, what is the ratio of the new force to the original force?

- A) $\frac{2}{9}$ B) $\frac{4}{9}$ C) $\frac{2}{3}$ D) $\frac{4}{3}$ E) NOTA

15. What is the coefficient of x^{16} in the expansion of the following expression?

$$(x^2 + 2x)^{10}$$

- A) 210 B) 960 C) 6,720 D) 8,064 E) NOTA

16. A dartboard is outlined by the graph of $9x^2 + 4y^2 = 36$. If you randomly hit the dartboard, what is the probability you are inside the bull's eye bounded by $x^2 + y^2 = 1$?

- A) $\frac{1}{6}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{2}{3}$ E) NOTA

17. Find:

$$\frac{(10 + 4i)(3 - 6i)}{3 + 9i}$$

- A) $-\frac{7}{5} - \frac{19}{5}i$ B) $4 + 6i$ C) $-3 - 7i$ D) $18 - \frac{16}{3}i$ E) NOTA

18. Find y.

$$\begin{bmatrix} -1 & -2 & 3 \\ 4 & 3 & 0 \\ 2 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$$

- A) $-\frac{13}{9}$ B) $-\frac{3}{4}$ C) $\frac{9}{16}$ D) $\frac{11}{16}$ E) NOTA

19. A school brings 6 theta, 9 alpha, and 16 calculus students to the state convention. If 4 students from the school are randomly selected to work on the poster, what is the probability that at least two divisions are represented? (Round to three significant digits.)

- A) 0.0623 B) 0.0695 C) 0.930 D) 0.938 E) NOTA

20. Simplify:

$$\log_4 5 + \log_2 25$$

- A) $\frac{2}{5} \log_5 2$ B) $\frac{2}{5} \log_2 5$ C) $\frac{5}{2} \log_5 2$ D) $\frac{5}{2} \log_2 5$ E) NOTA

21. Let $f(x) = 2x^3 + ax^2 + bx + c$ where a , b , and c are real numbers. The sum and the product of the roots of $f(x)$ are 6 and -5 , respectively. Given that 2 is a root of $f(x)$, what is the value of b ?

- A) -22 B) -17.5 C) 11 D) 21 E) NOTA

22. Find the area of the triangle with vertices at the origin and the foci of $25x^2 + 9y^2 - 250x + 36y + 436 = 0$?

- A) 4 B) $\sqrt{34}$ C) 20 D) $5\sqrt{34}$ E) NOTA

23. A bag contains 6 identical red marbles, 4 identical blue marbles, and 8 identical purple marbles. In how many different distinguishable orders can you take all of the marbles out of the bag by removing one at a time?

- A) 9,189,180 B) 56,808,540 C) 133,899,480
D) 387,420,489 E) NOTA

24. Solve:

$$|2x - 5| \leq \frac{1}{2}$$

- A) $\left(\frac{9}{4}, \frac{11}{4}\right)$ B) $\left[\frac{9}{4}, \frac{11}{4}\right]$ C) $\left(-\infty, \frac{9}{4}\right) \cup \left(\frac{11}{4}, \infty\right)$
D) $\left(-\infty, \frac{9}{4}\right] \cup \left[\frac{11}{4}, \infty\right)$ E) NOTA

25. Given: $f(x) = x^3$
 $g(x) = x + 1$
Find: $(f \circ g)(2)$

- A) -2 B) 3 C) 9 D) 27 E) NOTA

26. What is the 5th term of an arithmetic sequence with a common difference of 5 and first term of 2?

- A) 13 B) 15 C) 22 D) 27 E) NOTA

27. Given that $f(x+1)f(x) = x$ for all positive integers x . What is the ten thousandths digit of $f(2001)f(2004)$?

- A) 1 B) 5 C) 7 D) 9 E) NOTA

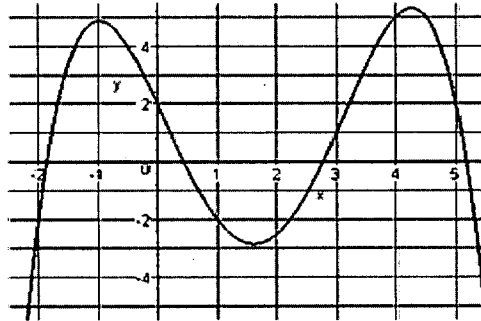
28. What is the probability of rolling a prime number on a fair six-sided die?

- A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{5}{6}$ D) 1 E) NOTA

29. What is the slope between $(-2,6)$ and $(c,3)$?

- A) $\frac{-c-2}{3}$ B) $\frac{-3}{c+2}$ C) $\frac{3}{c+2}$ D) $\frac{c+2}{3}$ E) NOTA

30. What is the sum of the coefficients excluding the constant term of the polynomial graphed below? (The polynomial has integral values at $x = -2, 0, 1, 3, 4,$ and 5 .)



- A) -4 B) -2 C) 0 D) 2 E) NOTA