MAΘ 1991 ALGEBRA 2 INDIVIDUAL

8

SOLUTIONS

1. $4^a \cdot 8^{-a} = 2^{2a} \cdot 2^{-3a} = 2^{-a} = \frac{1}{2^a} \Rightarrow$ (A)

2. $b = \frac{ac^n - a}{c-1} = \frac{a(c^n-1)}{c-1} \Rightarrow$ (A)

3. $V = 4(x-8)^2 = 4(x^2 - 16x + 64)$

$V = 4x^2 - 64x + 256 = 900$

$\therefore x^2 - 16x + 64 = 225$

$x^2 - 16x - 161 = 0 \longrightarrow (x-23)(x+7) = 0$

$x = 23$ in. $\Rightarrow$ (C)

4. let $x^2 = x^2 + 2$ in $P(x^2-1)$

$\therefore P(x^2+1) = (x^2+2)^2 + 5(x^2+2) + 3$

$= x^4 + 4x^2 + 4 + 5x^2 + 10 + 3$

$= x^4 + 9x^2 + 17 \Rightarrow$ (A)

5. $f(x) = \frac{1}{x},\ x \neq 0 \qquad g(x) = \frac{1}{x^2 + x - 6}$

$g(x) = \frac{1}{(x+3)(x-2)},\ x \neq -3, 2$

$\therefore$ (E)

6. $\frac{12!}{9!\,3!} x^9 (-2y)^3 = \frac{\cancel{12}^{2} \cdot 11 \cdot 10 \cdot 9!}{9! \cdot 3 \cdot 2 \cdot 1} x^9 (-8y^3)$

$= -1760 x^9 y^3 \Rightarrow$ (B)

7.
$$A^T = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} \longrightarrow \text{adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & -4 \end{bmatrix}$$

$a_{33} = -4 \Rightarrow$ (D)

8.

$i^{434} = i^{432+2} = i^{4n+2} = -1$

$i^{431} = i^{428+3} = i^{4n+3} = -i$

$\therefore i^{434} - i^{431} = -1 - (-i) = -1 + i \rightarrow$

$= i - 1$ (D)

9. $m = \frac{3-7}{1+2} = -\frac{4}{3} \longrightarrow m_\perp = \frac{3}{4} \Rightarrow$ (C)

10. $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^{n-1} \longrightarrow S = \frac{1}{1-1/3} = \frac{1}{2/3} = \frac{3}{2} \Rightarrow$ (C)

11. $S = \frac{50}{2}\left(2(1) + (50-1)\times 2\right) = 2500$

or $50^2 = 2500 \Rightarrow$ (C)

12. ${}_4P_2 = \frac{4!}{2!} = \frac{4\cdot 3\cdot \cancel{2!}}{\cancel{2!}} = 12$

$12 + 1 = 13 \Rightarrow$ (B)

13.

(A) trivially false since $x+2$ is not a factor of $(a+b)$ $\therefore$ FALSE

(B) $\sqrt{x^2} = |x| = \pm x$, and $x \neq -x$ $\therefore$ FALSE

(D) let $x = -2$, then $-(-2) = +2$ $\therefore$ FALSE

(C) $\frac{a+b}{a-b} = -\frac{(a+b)^2}{b^2-a^2} = \frac{(a+b)\cancel{(a+b)}}{\cancel{(a+b)}(a-b)} = \frac{a+b}{a-b}$

for $a^2 \neq b^2$, and $a \neq b$ TRUE

$\Longrightarrow$ (C)

14. i is a root $\Rightarrow$ $-i$ is also a root, $(x+i)$ & $(x-i)$ are factors $\Rightarrow$ x^2+1 is a factor

$\frac{x^4-2x^3-2x-1}{x^2+1} = x^2-2x-1$

$x = \frac{2 \pm \sqrt{4+4}}{2} = \frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2} \Rightarrow$ (C)

15.

all factorials after $5! = 120$ end in $0 \Rightarrow$ (E)

16. $-2 = 128r^3 \longrightarrow r^3 = -\frac{1}{64},\ r = -\frac{1}{4}$

$\therefore$ G.M. ARE: $128\left(-\frac{1}{4}\right) = -32$

$-32\left(-\frac{1}{4}\right) = 8$

SUM $= -32 + 8 = -24 \Longrightarrow$ (A)

18. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$a = 4,\ b = 3,\ \therefore A = \pi(4)(3) = 12\pi$

(A)

17. $\log x^{\log x} = \log \frac{x^4}{10^4} = \log x^4 - \log 10^4$

$\therefore (\log x)^2 = 4\log x - 4 \longrightarrow (\log x)^2 - 4\log x + 4 = 0$

and $(\log x - 2)^2 = 0 \longrightarrow \log x = 2 \longrightarrow x = 100$

$\Longrightarrow$ (C)

19. HARMONIC MEAN $= \frac{2(70)(55)}{70+55} = 61.6 \Longrightarrow$ (A)

20.

$P = \frac{\cancel{9}^{\,3}}{\cancel{15}_{\,5}} \cdot \frac{\cancel{8}^{\,4}}{\cancel{14}_{\,7}} = \frac{12}{35} \Longrightarrow$ (C)

21. $4(x^2 - 2x + 1) + 4(y^2 + y + \frac{1}{4}) = 27 + 5$

$4(x-1)^2 + 4(y + \frac{1}{2})^2 = 32$

$(x-1)^2 + (y + \frac{1}{2})^2 = 8 = (2\sqrt{2})^2$

$\therefore R = 2\sqrt{2} \Longrightarrow$ (D)

22. let x = amt. of salt to be added

$6 + x = .2(60 + x) \longrightarrow 6 + x = 12 + .2x$

$.8x = 6 \longrightarrow x = \frac{6}{.8} = \frac{60}{8} = 7\frac{1}{2}$ lb. $\Longrightarrow$ (A)

23. $\log_4 \frac{|2x+5|}{|3x+1|} = \frac{1}{2}$, since $\frac{|2x+5|}{|3x+1|}$ is always > 0

$\frac{2x+5}{3x+1} = 4^{1/2} = 2 \longrightarrow 2x + 5 = 6x + 2 \longrightarrow 4x = 3$

$x = \frac{3}{4} \Longrightarrow$ ~~D~~ E

also $-7/8$

24. $P(2) = 20 = 32 - 12 + 2A + B \rightarrow 2A + B = 0$

$P(-2) = 24 = 32 - 12 - 2A + B \rightarrow -2A + B = 4$

$2B = 4$

$\Rightarrow$ (C) $\qquad B = 2 \rightarrow A = -1$

25.

$a_{11} = 1 = 3 + (5-1)d \rightarrow d = -\frac{1}{2}$

$\therefore a_7 = 3 = a_1 + (7-1)(-\frac{1}{2}) \rightarrow a_1 - 3 = 3,\ a_1 = 6$

$\Rightarrow$ (A)

26. $x^5 + 0x^4 - 8x^3 - 6x^2 + 7x + 6 = 0$

$\therefore$ SUM $= 0 \Rightarrow$ (B)

27.

$$A^{-1} = -\frac{1}{8}\begin{bmatrix} 0 & 4 \\ 2 & 6 \end{bmatrix} = \begin{bmatrix} 0 & -1/2 \\ -1/4 & -3/4 \end{bmatrix} \Rightarrow \text{(E)}$$

$A \cdot A^{-1} = I$

28. $$\frac{9!}{3!\,2!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4}^{2} \cdot \cancel{3!}}{\cancel{3!}\,\cancel{2!}} = 30240 \text{ ways.}$$

$\Rightarrow$ (B)

29. $A' = \{2, 4, 6, 8, 10\}$, $B' = \{1, 3, 5, 7, 9\}$

$A' \cap C = \{4, 10\}$, $B' \cap C = \{1, 7\}$

$(A' \cap C) \cup (B' \cap C) = \{1, 4, 7, 10\} = C$

$\Rightarrow$ (D)

30. $$\frac{5+2i}{6-i} \cdot \frac{6+i}{6+i} = \frac{28+17i}{36+1} = \frac{28+17i}{37} \Rightarrow \text{(D)}$$