Algebra I Team Answers Regional Competition March 14, 1992

1. $-3/4$
2. $3\ 1/3$ or $\frac{10}{3}$
3. -3
4. $\frac{27x^{18}y^{-3}}{64y^{12}}$ $\frac{27}{64} x^{18} y^{-12} z^{3}$
5. 4
6. $-1/4$
7. 99
8. 1
9. 2616
10. 78
11. 5
12. 0
13. -3
14. $2(x + 1)(x - 1)(4x + 3)$
15. 4

① $-2^2 + 10\cdot 2^{-3} + (-1)^{50} + 4^0$

$= -4 + \frac{10}{8} + 1 + 1$

$= -2 + \frac{5}{4} = \boxed{\frac{-3}{4}}$

② $|5-3x| = 14$

$5-3x = 14$ OR $5-3x = -14$

$-3x = 9$ $\quad -3x = -19$

$x = -3$ $\quad x = \frac{19}{3} = 6\frac{1}{3}$

SUM $= -3 + 6\frac{1}{3} = \boxed{3\frac{1}{3}}$

③ $m_{AB} = \frac{-1-(-3)}{2-0} = 1$

$m_{AC} = \frac{3-(-3)}{-2-0} = -3$

$m_{BC} = \frac{3-(-1)}{-2-2} = -1$

SUM $= 1 + (-3) + (-1) = \boxed{-3}$

④ $\left(\frac{18xy^{-3}z^2}{24x^{-5}yz}\right)^3$

$= \left(\frac{3x^6z}{4y^4}\right)^3$

$= \frac{3^3(x^6)^3z^3}{4^3(y^4)^3}$

$= \boxed{\frac{27x^{18}z^3}{64y^{12}}}$

⑤ $\sqrt{2}(\sqrt{8}+4)(\sqrt{2}-1)$

$= \sqrt{2}(\sqrt{16} - \sqrt{8} + 4\sqrt{2} - 4)$

$= \sqrt{2}(4 - 2\sqrt{2} + 4\sqrt{2} - 4)$

$= \sqrt{2}(2\sqrt{2}) = 2(\sqrt{2})^2 = \boxed{4}$

⑥ $2(1-3x) = 4x - 2(x+1)$

$2 - 6x = 4x - 2x - 2$

$2 - 6x = 2x - 2$

$4 = 8x$

$x = \frac{4}{8} = \frac{1}{2}$

$-4(2y+1) = 7 + 2y - (2y+1)$

$-8y - 4 = 7 + 2y - 2y - 1$

$-8y - 4 = 6$

$-8y = 10$

$y = \frac{-10}{8} = \frac{-5}{4}$

$7z - 5 = 4(1-3z) + z$

$7z - 5 = 4 - 12z + z$

$7z - 5 = 4 - 11z$

$18z = 9$

$z = \frac{9}{18} = \frac{1}{2}$

$\therefore\ x+y+z = \frac{1}{2} + \left(\frac{-5}{4}\right) + \frac{1}{2} = \boxed{\frac{-1}{4}}$

⑦ $\frac{(a+b)^2(a-b)^2}{(a+b)(a-b)} = (a+b)(a-b)$

$= (10+1)(10-1)$

$= (11)(9)$

$= \boxed{99}$

⑧ Expanding obtain

$8x^2-10x-3 + 2x^2+5x-3 = 6x^2+5x-6 + 4x^2-6x-4$

$10x^2 - 5x - 6 = 10x^2 - x - 10$

$-4x = -4$

$\boxed{x = 1}$

⑨ $A = 2\cdot5\cdot23 = 2310$

$12 = 2^2\cdot3 \quad 100 = 2^2\cdot5^2$

$B = 2^2\cdot3\cdot5^2 = 300$

$120 = 2^3\cdot3\cdot5 \quad 126 = 2\cdot3^2\cdot7$

$C = 2\cdot3 = 6$

$A+B+C = 2310+300+6 = \boxed{2616}$

⑩ $5n + 10d + 25q = 1230$

$q = 2n$

$d = n+6$

$5n + 10(n+6) + 25(2n) = 1230$

$5n + 10n + 60 + 50n = 1230$

$65n = 1170$

$n = 18$

$q = 2(18) = 36$

$d = 18+6 = 24$

TOTAL # COINS $= 18+36+24 = \boxed{78}$

⑪ L_1: $y-(-1) = 3(x-4)$

$y = 3x-13$

L_2: $y = -x+5$

Equating y obtain

$3x-13 = -x+5$

$4x = 18$

$x = \frac{9}{2} \longrightarrow y = -\frac{9}{2}+5 = \frac{1}{2}$

So $x+y = \frac{9}{2}+\frac{1}{2} = \boxed{5}$

OR simply observe L_2 is the line $x+y=5$!

⑫ $|5-2x| < 8$

$-8 < 5-2x < 8$

$-13 < -2x < 3$

$\frac{13}{2} > x > \frac{-3}{2}$

Integer solutions are $-1, 0, 1, 2, 3, 4, 5,$ and $\boxed{\text{Product} = 0}$ since 0 is a factor

⑬ $(2n)^2 - 7 = 2n+5$

$4n^2-2n-12 = 0$

$2n^2-n-6 = 0$

$(2n+3)(n-2) = 0$

$n = \frac{-3}{2}$ or $n = 2$

PRODUCT $= \left(\frac{-3}{2}\right)(2) = \boxed{-3}$

⑭ $8x^3+6x^2-8x-6$

$= 2x^2(4x+3) - 2(4x+3)$

$= 2(x^2+1)(4x+3)$

$= \boxed{2(x+1)(x-1)(4x+3)}$

⑮ let $w = 1/y$

$2x-3w = 1 \longrightarrow 4x-6w = 2$

$3x+6w = 5 \qquad 3x+6w = 5$

$7x = 7$

$x = 1$

$2(1)-3w = 1$

$-3w = -1$

$w = 1/3 \longrightarrow y = 3$

$x+y = 1+3 = \boxed{4}$