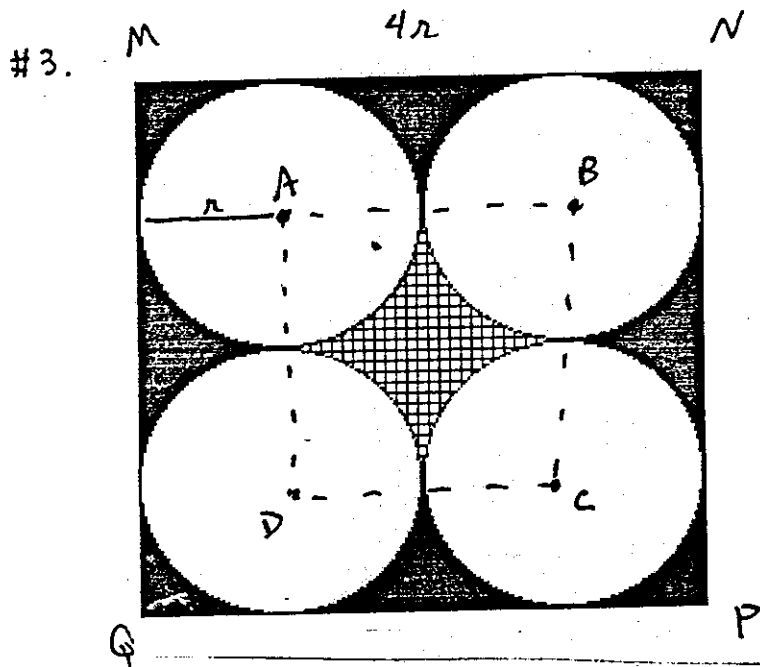


SOLUTIONS
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#1. $x^4(x+3) - 3x^2(x+3) - 4(x+3)$ -3
 $(x+3)(x^4 - 3x^2 - 4) = (x+3)(x^2-4)(x^2+1)$
 $x = -3, \pm 2, \pm i$

#2 $A \cos \theta + B \sin \theta = C \cos(\theta - \phi)$, $C = \sqrt{A^2 + B^2}$, $\sin \phi = \frac{B}{C}$
 $\cos \phi = \frac{A}{C}$
 $C = 2$ $\phi = -30^\circ$
 $2 \cos(\theta - (-30)) = \sqrt{2}$
 $\cos(\theta + 30) = -\frac{\sqrt{2}}{2}$
 $\theta + 30 = 135^\circ$ $\theta + 30 = 225^\circ$
 $\therefore \theta = 105^\circ \text{ \& } 195^\circ$

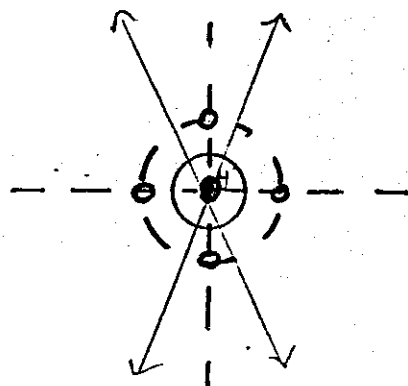


Checked area = $ABCD - (4 \cdot \frac{1}{4} \pi r^2)$
 $= 4r^2 - \pi r^2$

Black area = $MNPQ - [ABCD + 4(\frac{3}{4} \pi r^2)]$
 $= 16r^2 - (4r^2 + 3\pi r^2)$

Ratio $\frac{4r^2 - \pi r^2}{12r^2 - 3\pi r^2} = \frac{1}{3}$

#4.

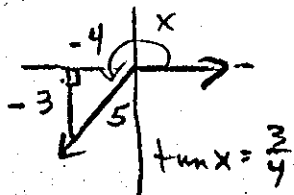


5 points

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#5. $\tan 3x = \tan(x+2x) = \frac{\tan x + \tan 2x}{1 - \tan x \cdot \tan 2x}$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$



$$\tan 2x = \frac{\frac{3}{2}}{1 - \frac{9}{16}}$$

$$\tan 2x = \frac{24}{7}$$

$$= \frac{\frac{3}{4} + \frac{24}{7}}{1 - \frac{3}{4} \cdot \frac{24}{7}} \cdot \frac{28}{28}$$

$$= \frac{21 + 96}{28 - 72} = \boxed{\frac{117}{44}}$$

#6 ΔABC sides 4, 13, 15 = $\sqrt{16(12)(3)(1)} = 24$

ΔABD sides 4, 19, 21 = $\sqrt{22(18)(3)(1)} = 6\sqrt{33}$

ΔADC sides 19, 20, 15 = $\sqrt{27(8)(7)(12)} = 36\sqrt{14}$

ΔBCD sides 13, 20, 21 = $\sqrt{27(14)(7)(6)} = 126$

$$\boxed{150 + 6\sqrt{33} + 36\sqrt{14}}$$

#7.

$$X^2 = X + \sqrt{X + \sqrt{X + \sqrt{X + \dots}}}$$

$$X^2 = X + X$$

$$X^2 = 2X$$

$$X^2 - 2X = 0$$

$$X(X-2) = 0$$

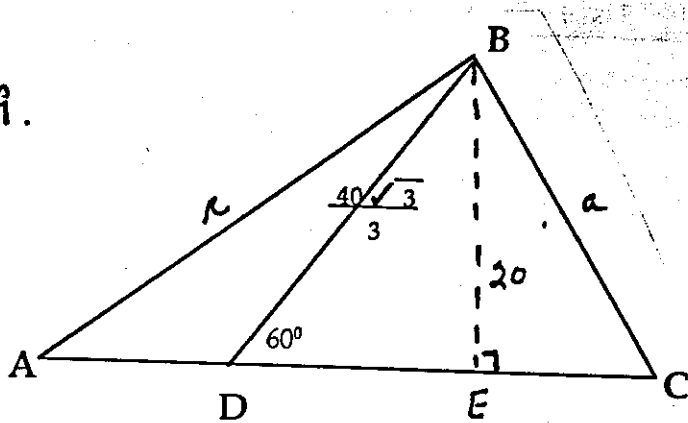
$$X = 0, 2$$

$$\boxed{0, 2}$$

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8 $(2+2i)^7 = (2\sqrt{2} \operatorname{cis} 45^\circ)^7 = 1024\sqrt{2} \operatorname{cis} 315^\circ = \boxed{1024 - 1024i}$

9.



$$\sin C = \frac{4}{5} = \frac{20}{a} \quad \therefore a = 25$$

$$\sin A = \frac{5}{13} = \frac{20}{c} \quad \therefore c = 52$$

$$EC = 15 \text{ \& } AE = 48 \quad \therefore b = \underline{63}$$

140

#10. $64 \cdot 64^{\frac{1}{3}} \cdot 64^{\frac{1}{9}} \cdot 64^{\frac{1}{27}} \dots$

$$64^{1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots}$$

$$64^{\frac{3}{2}} = \boxed{512}$$

$$S = \frac{a}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

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#11.

$$\begin{aligned} a + b + 2c &= 2 & \rightarrow & 3a + 3b + 6c = 6 \\ 2a - 3b + 4c &= -11 & \rightarrow & 2a - 3b + 4c = -11 \\ 3a - 5b - 6c &= 0 & & 5a + 10c = -5 \\ & & & a + 2c = -1 \quad 2 + 2c = -1 \\ & & & 2c = -3 \\ & & & c = -\frac{3}{2} \\ & & & -4a - 2c = -5 \quad 2c = -3 \\ & & & -4a = -5 + 3 \\ & & & -4a = -2 \\ & & & a = \frac{1}{2} \quad x = \frac{1}{2} \quad z = -\frac{2}{3} \\ & & & -3a = -6 \\ & & & a = 2 \end{aligned}$$

$$\begin{aligned} & \rightarrow 3a - 5b - 6c = 0 \\ & \rightarrow 5a + 5b + 10c = 10 \\ & \underline{8a + 4c = 10} \end{aligned}$$

$$2 + b + 2\left(-\frac{3}{2}\right) = 2$$

$$b = 3 \quad y = \frac{1}{3} \quad \frac{1}{3} + \frac{-2}{3} + \frac{1}{2} = \boxed{\frac{1}{6}}$$

#12.

$$\log_9 \frac{1}{2} \cdot \log_2 3$$

$$\frac{1}{2 \log_9 \frac{1}{2}} \cdot \frac{\log_2 3}{1}$$

$$\frac{\log_3 \frac{1}{2}}{2} \cdot \frac{\log_2 3}{1}$$

$$\frac{\log_3 1 - \log_3 2}{2} \cdot \frac{\log_2 3}{1}$$

$$\frac{0 - \log_3 2}{2} \cdot \frac{1}{\log_3 2}$$

$$\boxed{-\frac{1}{2}}$$