Precalculus Answers:

January Regional

TEAM

1. 154
2. $\frac{20}{9}$ or $2\frac{2}{9}$ or $2.\overline{2}$
3. 5.6
4. $\frac{-\sqrt{5}}{3}$
5. 35.3
6. 95
7. $328\sqrt{2}$
8. 36
9. 7
10. 12
11. $\frac{16}{5}$ or $3\frac{1}{5}$ or 3.2
12. 344
13. $\frac{118+2\sqrt{2}}{3}$
14. $\frac{16\sqrt{2}}{3}$
15. -1

INDIVIDUAL

1.	B.	16.	E
2.	D	17.	B
3.	D	18.	C
4.	D	19.	A
5.	A	20.	A
6.	C	21.	C
7.	A	22.	C
8.	B	23.	A
9.	A	24.	C
10.	B	25.	C
11.	B	26.	B
12.	A	27.	C
13.	B	28.	C
14.	A	29.	D
15.	A	30.	A

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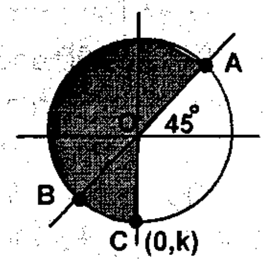
Solutions:

1. Using the quadratic formula we get $\frac{2\pm\sqrt{12}}{2} = 1 \pm \sqrt{3}$ which is choice **B**.
2. One person per house per 4 days, gives 16 people for 16 houses per 4 days. So 16 people can paint 8 houses in 2 days. **D**.
3. The ranges of the Arcfunctions are limited. Arccotangent has a range of $(0, \pi)$, so choice **D** cannot be true.
4. $2i \bullet i\sqrt{2} = -2\sqrt{2}$ which is choice **D**.
5. $1 + \sqrt{\frac{x}{4}} = 36$, $\sqrt{\frac{x}{4}} = 35$, $\frac{x}{4} = 1225$ and so $\sqrt{x} = \sqrt{1225 \bullet 4} = 35 \bullet 2 = 70$. Choice **A**.
6. $\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = 5$ so $\frac{3}{ab} = 5$ and $5ab = 3$ and $ab = \frac{3}{5}$. Choice **C**.
7. $\frac{9}{(i-1)^2} = \frac{9}{-2i} = \frac{9}{-2i} \bullet \frac{i}{i} = \frac{9i}{2}$. Choice **A**.
8. $b^2 - 4ac \le 0$, $16 - 4k \le 0$, $-4k \le -16$, $k \ge 4$, Choice **B**.
9. $\sqrt{2}\,(leg) = hypotenuse$ so $\sqrt{2}\sec\theta = \sqrt{3}\tan\theta$ and $\frac{\sqrt{2}}{\cos\theta} = \frac{\sqrt{3}\sin\theta}{\cos\theta}$ and $\sin\theta = \frac{\sqrt{2}}{\sqrt{3}}$. So $\theta \approx 54.7°$ which is choice **A**.
10. $[e] = 2$, $[-\pi] = -4$, $[0] = 0$. The sum is -2 which is choice **B**.
11. $\sin\theta(2\sin\theta - 1) = 0$, so $\sin\theta = 0$ or $\sin\theta = \frac{1}{2}$. Solutions are $0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$, or $\frac{0\pi}{6}, \frac{6\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$ so a=0, b=1, c=5, d=6. b+d= 7 which is choice **B**.
12. $x^{\frac{3}{5}} + a = 1$, $x^{\frac{3}{5}} = 1 - a$, $x = (1-a)^{\frac{5}{3}}$ which is choice **A**.
13. The sum will be $\frac{a_1}{1-r} = \frac{\sqrt{3}}{1-\frac{1}{\sqrt{3}}} = \frac{3}{\sqrt{3}-1}$ when multiplied by $\frac{\sqrt{3}}{\sqrt{3}}$. Using the conjugate we multiply by $\frac{\sqrt{3}+1}{\sqrt{3}+1}$ which gives $\frac{3\sqrt{3}+3}{2}$. Choice **B**.
14. $(x-5) = -a(y+3)^2$ and $\frac{1}{4a} = 3$ so $a = \frac{1}{12}$ using the intercept $A - 5 = \frac{-1}{12}(0+3)^2$ which gives $A = \frac{17}{4}$ so $4A = 17$. Choice **A**.
15. Using the change of base rule, $\frac{\log 3}{\log 2} \bullet \frac{\log 4}{\log 3} \bullet \frac{\log 5}{\log 4} \bullet \ldots \bullet \frac{\log 11}{\log 10}$ which reduces to $\frac{\log 11}{\log 2} = \log_2 11$, choice **A**.
16. Using the Triangle Inequality Theorem, $0 < AB < 20$ so all can be possibilties for AB. Choice **E**.
17. $\frac{225}{360}(16\pi)$ since the area of the circle of radius 4 is 16π. The answer is 10π which is choice **B**.



18. One of the acute angles x, can be found by $\cos x = \frac{5}{12}$ and the other by $90° - Cos\frac{5}{12}$. These angles are approximately 65.37568 and 24.6243 which gives a difference of 40.8°, choice **C**.
19. The roots are $1+\sqrt{2}, 1-\sqrt{2}, i, -i$ and the first two give a sum of 2 and a product of -1. This gives quadratic $x^2 - 2x - 1$. The second roots give a sum of 0 and a product of 1. This gives quadratic $x^2 + 1$. Multiplying the two quadratics gives $x^4 - 2x^3 - 2x - 1$ so B= 0, choice **A**.
20. The volume of the pool is $\pi r^2 h = 1800\pi$ cubic feet. Since the water is going in at a rate of 1 cu. foot per hour, it will take approx. 5654.8668 hours, or choice **A**.
21. The equation of the ellipse described is $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and so $A = \frac{1}{16}, B = \frac{1}{9}$ and the sum is $\frac{25}{144}$ which is choice **C**.

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<u>Solutions</u> (page two of two):

22. Dividing by 225 gives $\frac{x^2}{9}+\frac{y^2}{25}=1$ and and this tells us that the region traces out a circular region of radius 5 and area 25π which is choice ***C***.

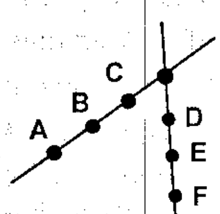
23. For the interval given, the first absolute value expression is equivalent to $12-3x$ and the second is equivalent to $5x+10$. The total expression is $2x+22$ so the sum of A and B is 24, choice ***A***.

24. $5x+4x+3x=1800$ gives that the original deal was for contributions of (x=1500) \$7500, \$6000 and \$4500. Since we do not know which person did not contribute all of their agreement, we could have \$8000, \$6500, \$3500 or \$8000, \$5000, \$5000 or \$6500, \$6500, \$5000. So the only one of the choices which could not be a contribution is \$7500. Choice ***C***.

25. The prob. of reaching D is $\frac{8}{20}$, and the prob. of reaching F is $\frac{1}{4}$. The product would be $\frac{1}{10}$ which is choice ***C***.

26. Setting $\frac{7}{6}=1+\frac{1}{x}$ and solving gives x=6. Doing the same respectively gives $\frac{7}{8}$, 3 and 20. Since x>1, choice B cannot be a value for y. Choice **B**.

27. The lines in question (see picture) are $\overline{DC},\overline{DB},\overline{DA}$, $\overline{EC},\overline{EB},\overline{EA}$ $\overline{FC},\overline{FB},\overline{FA}$, 9 lines, choice ***C***.



28. Using the law of cosines, we get $36=64+144-2(8)(12)\cos C$ which gives *C* is approximately 26.384 degrees. Using the law of cosines in the right-side triangle we get the median is $\sqrt{14}$ in length. Using the law of sines we get $\frac{\sin x}{8}=\frac{\sin 26.384}{\sqrt{14}}$ but since this is an SSA situation, the angle BDC may be 71.83 or 180-71.83 degrees. Since the Hinge theorem says that $\angle BDA<\angle BDC$ we know that $m\angle BDC\approx 108.2°$, choice ***C***.

29. Let the original radius be r and the new radius be R. $C=2\pi r=40\pi$ so the new circumference is $40\pi-5$ which is equal to $2\pi R$ so $R=\frac{40\pi-5}{2\pi}=20-\frac{5}{2\pi}$. The area goes from πr^2 which is 400π to $\pi(20-\frac{5}{2\pi})^2$ which gives a difference of $400\pi-\pi(400-\frac{100}{\pi}+\frac{25}{4\pi^2})$ or $100-\frac{25}{4\pi}$. So the percent of change is $100(100-\frac{25}{4\pi})$ divided by 400π which is $25(\frac{1}{\pi}-\frac{1}{16\pi^2})$ or choice **D**.

30. The graphs of $x-\sqrt{3}\,y=0$ and $x+y=2\sqrt{3}$ intersect where $y+\sqrt{3}\,y=2\sqrt{3}$ (subtract the two eqns) so $y(1+\sqrt{3})=2\sqrt{3}$ and $y=\frac{2\sqrt{3}}{1+\sqrt{3}}$ which is the height of the triangle in question. The second equation has x-intercept $(2\sqrt{3},0)$ so the base of the triangle is $2\sqrt{3}$. The area is $\frac{1}{2}(2\sqrt{3})(\frac{2\sqrt{3}}{1+\sqrt{3}})$. which equals $\frac{6}{1+\sqrt{3}}$. Multiply by $\frac{1-\sqrt{3}}{1-\sqrt{3}}$ gives $\frac{6-6\sqrt{3}}{-2}$ or $3\sqrt{3}-3$. Choice ***A***.