Pre Calculus Individual - Solutions Part I Jan 8, 1994

(20)

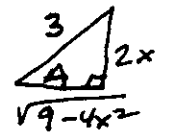
A ① $x^2-6x+9+y^2+4y+4 = 4+9+4$
$\pi r^2 = 17\pi$

② $x^2-2x-3 \geq 0$; $(x-3)(x+1) \geq 0$; $x \geq 3$ or $x \leq -1$

B ③ $g(-1)$ is <0 $f(a)$ where $a<0$ is in QII

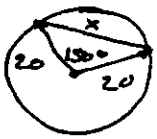
B ④ $\frac{5\pi}{9} \cdot \frac{180}{\pi} = 100$

A ⑤ $\sin\theta - \cos\theta = \frac{-3}{5} - \frac{-4}{5} = \frac{1}{5}$

C ⑥ 

$\tan A = \frac{2x}{\sqrt{9-4x^2}}$
$9-4x^2>0 \Rightarrow 4x^2-9<0 \Rightarrow$
$-3/2 < x < 3/2$

C ⑦ $x^2 = 20^2+20^2-2\cdot20\cdot20\cos150°$
$x^2 = 800 - 800\cdot-\sqrt{3}/2$
$x^2 = 800+400\sqrt{3}$
$= 400(2+\sqrt{3})$
$x = 20\sqrt{2+\sqrt{3}}$



C ⑧ $AD\cdot DB = CD^2$
$4\cdot9 = CD^2$
$CD = 36$
$CD = 6$
$\tan\angle CAB = \frac{6}{4} = 3/2$
$\angle CAB = 56°$

C ⑨ $\frac{\tan205°-\tan115°}{\tan245°+\tan335°} = \frac{\tan25°-(-\cot25°)}{\cot25°+-\tan25°}$
$= \frac{a+\frac{1}{a}}{\frac{1}{a}-a} = \frac{a^2+1}{1-a^2}$

A ⑩ $\sin2\theta = \cos\frac{1}{2}\theta \Rightarrow \sin(90\pm\frac{1}{2}\theta) = \sin2\theta$
$90°\pm\frac{1}{2}\theta = 2\theta+360°K$

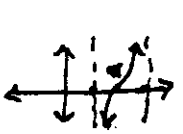
$\frac{3}{2}\theta = 90°+360°K$
$3\theta = 180°+720°K$
$\theta = 60°+240°K$
$\theta = 60°, 300°$

OR
$\frac{5}{2}\theta = 90°+360°K$
$5\theta = 180°+720°K$
$\theta = 36°+144°K$
$\theta = 36°, 180°, 324°$

$60+300+36+180+324 = 900°$

B ① $\frac{\sin(\frac{\pi}{2}+\theta)+\cos(\pi+\theta)}{\sin(\frac{3\pi}{2}-\theta)-\cot(-\theta)} = \frac{\cos\theta-\cos\theta}{-\cos\theta+\cot\theta}$

$=0$

D ⑫ 

C ⑬ $6\sin x\cos x = 3\sin2x$ $P = \frac{2\pi}{2} = \pi$

D ⑭ $\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta} = \frac{\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$
$= \frac{1+\sin\theta}{\cos\theta(1+\sin\theta)} = \frac{1}{\cos\theta} = \sec\theta$

D ⑮ $\sin\frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1-\frac{b}{c}}{2}}$
$= \sqrt{\frac{c-b}{2c}}$

B ⑯ $\sin(\alpha+\beta) = \sin\alpha\cos\beta+\cos\alpha\sin\beta =$
$\frac{12}{13}\cdot\frac{3}{5}+\frac{5}{13}\cdot\frac{4}{5} = \frac{56}{65}$ $56+65=121$

B ⑰ $2\sec x = \tan x + \frac{1}{\tan x}$
$2\sec x = \frac{\tan^2x+1}{\tan x} \Rightarrow 2\sec x = \frac{\sec^2x}{\tan x}$
$\Rightarrow \sec x = 0$ OR $2\tan x = \sec x \Rightarrow 2\frac{\sin x}{\cos x} = \frac{1}{\cos x}$
$\Rightarrow 2\sin x = 1 \Rightarrow \sin x = 1/2 \Rightarrow x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$
sum is π

C ⑱ $2x=4$ and $-y=3$; $x=2$ and $y=-3$
$x+y=-1$

A ⑲ $P = \frac{\text{area } ① \text{ with radius } \frac{1}{2}}{\text{area square}} = \pi/4$

A ⑳ $-b\pm\sqrt{b^2-4ac}$ is same for both equations
$\frac{1}{4}/\frac{1}{8} = 8/4 = 2$

B ㉑ $\frac{\frac{18}{1}\cdot\frac{19}{2}\cdot\frac{20}{3}\cdots\frac{36}{19}}{\frac{20}{1}\cdot\frac{21}{2}\cdot\frac{22}{3}\cdots\frac{36}{17}} = \frac{\frac{36!}{17!\,19!}}{\frac{36!}{19!\,17!}} = 1$

D ㉒ $\frac{1}{\log_N ab} = \frac{1}{\log_N a+\log_N b}$

D ㉓ $3*48 = \sqrt{3\cdot48} = 12$ $12*9 = \sqrt{12\cdot9} = 6\sqrt{3}$

A ㉔ $1-5x5$; $4-4x4$; $9-3x3$; $16-4x4$; $25-5x5$
$1+4+9+16+25 = 55$

C ㉕ $\frac{266\rho}{2000\theta} = r^3$ $r^3 = \frac{133}{1000}$ $r = \frac{11}{10} = 1+i$

A ㉖ $(2x+1)(x-2) = 0$ at $-1/2$ and 2

E ㉗ $x-3 = 2-2\sqrt{2x}+x$; $5/2 = \sqrt{2x}$; $\frac{25}{4} = 2x$; $x = \frac{25}{8}$

C ㉘ $2^{1/2} \div 2^2 = 2^{-3/2} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}$

A ㉙ (I) $A-\emptyset = A\cap U = A$ YES (II) $A\cap(B\cap C')' =$
III $A\cap(B\cup B') = A\cap U = A$ YES $A\cap(B'\cup C) = (A\cap B')\cup(A\cap C)$ YE

㉚ $\sin5x = 0$
$5\cos^4x\sin x - 10\cos^2x\sin^3x + \sin^5x = 0$
D $\sin x[5\cos^4x - 10\cos^2x(1-\cos^2x)+(1-\cos^2x)^2]$
$5\cos^4x - 10\cos^2x + 10\cos^4x + 1 - 2\cos^2x + \cos^4x = 0$
$16\cos^4x - 12\cos^2x + 1 = 0$
$\cos^2x = \frac{12\pm\sqrt{80}}{32} = \frac{3\pm\sqrt{5}}{8}$ $\sqrt{\frac{3-\sqrt{5}}{8}}$ is in given interval
5+6 = 13