

**MU ALPHA THETA REGIONAL COMPETITION  
PRECALCULUS INDIVIDUAL TEST**

**JANUARY 23, 1993**

"NOTA" = None Of The Above

1) Evaluate the sum:  $\sum_{k=1}^{\infty} \frac{1}{k(k+1)(k+2)}$

- a) 0.22                      b) 0.25                      c) 0.33                      d) 0.5                      e) NOTA

2) Let  $M = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$

Find  $N = M^4 + M^3 + M^2 - M + I + M^{-1}$ . (Where I is the identity matrix). Then find the sum of the elements in N.

- a) 8                      b) 9                      c) 10                      d) 18                      e) NOTA

3) Let  $[ ]$  denote the greatest integer function.  
Evaluate:  $[-1.5] + [\pi^2] + [\ln(1.1)] + [1.999...] - [e]$

- a) 6                      b) 7                      c) 8                      d) 9                      e) NOTA

4) Given that  $\log_6(3) = A$  and  $\log_6(6) = B$ , find  $\log_6(2)$ .

- a)  $B + AB^2$                       b)  $B - AB$                       c)  $A^2 - AB$                       d)  $AB$                       e) NOTA

5) Given that :

$$\frac{4x+3y}{xy} = 5$$

$$\frac{3x-8y}{xy} = -\frac{13}{2}$$

Find the value of  $x + y$ .

- a) 3                      b)  $\frac{11}{3}$                       c)  $\frac{7}{2}$                       d)  $\frac{9}{2}$                       e) NOTA

- 6) When solving a system of three equations (in three variables,  $x$ ,  $y$ , and  $z$ ), a student used

Cramer's Rule and found the following for  $x$  :

$$x = \frac{\begin{vmatrix} -4 & 2 & -1 \\ 3 & 1 & 1 \\ 5 & 1 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & -1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix}}$$

Determine the value of  $z$ .

- a) -2                      b) 0                      c) 1                      d) 3                      e) NOTA
- 7) Given that  $\ln 3 = a$ ,  $\ln 5 = b$ , then solve for  $x$  if  $5^x = 3^{2x+1}$
- a)  $\frac{2b}{a}$                       b)  $\frac{a}{2b}$                       c)  $\frac{a}{(b-2a)}$                       d)  $\frac{b}{(a-2b)}$                       e) NOTA
- 8) On the interval  $[0,360^\circ)$ , where is  $\sin(x) > \cos(x)$  ?
- a)  $(45^\circ, 135^\circ]$                       b)  $(0^\circ, 45^\circ) \cup (135^\circ, 360^\circ)$
- c)  $(45^\circ, 225^\circ]$                       d)  $(0^\circ, 45^\circ) \cup (225^\circ, 360^\circ)$                       e) NOTA
- 9) Right triangle ABC has AB as the longest side and  $b = 88$ . If  $a, b, c$  are all integers and  $\text{GCF}(a, b, c) = 1$ , find the minimum perimeter of ABC.
- a) 330                      b) 396                      c) 462                      d) 469                      e) NOTA

- 10) The digital sum of a number is the sum of its digits. If the resulting sum is greater than 9, then the digits are summed again, and so on until a one digit number is obtained. Denote the digital sum of an integer  $n$  as  $\delta(n)$ .

Evaluate: 
$$\sum_{n=1}^{100} \delta(n)$$

- a) 450                      b) 470                      c) 495                      d) 496                      e) NOTA
- 11) Define a sequence recursively as  $a_{n+2} = 4a_{n+1} - 3a_n$ . If the first two terms of this sequence are  $a_0 = 2$  and  $a_1 = 4$ , find the least value of  $n$  such that  $a_n > 1000$ .
- a) 6                          b) 7                          c) 8                          d) 9                          e) NOTA
- 12) Two cars are traveling towards each other, one at a rate of 20 kmh, the other at 30 kmh. Olympic track star Linford Christie runs at a rate 40 kmh, and starts off next to the car traveling 20 kmh, and runs towards the other car. When he reaches the next car he immediately turns around and runs the other direction. If the cars are 50 km apart to begin, how far (in km) does Christie run before being smashed by the two cars?
- a) 25                          b) 35                          c) 40                          d) 50                          e) NOTA
- 13) If  $\cos(2A) = 0.6$  and  $A$  is an acute angle, find  $\sin^2(A)$ .
- a) 0.125                      b) 0.2                      c) 0.5                      d) 0.8                      e) NOTA
- 14) Let  $u=(1,-1)$ ,  $v=(3,-2)$ , and  $w=(1,-2)$ . If  $w = Au + Bv$ , where  $A$  and  $B$  are scalars, find  $A^2 + AB + B^2$ .
- a) 13                          b) 17                          c) 21                          d) 25                          e) NOTA
- 15) If  $f(x) = \sin(x) + \cos(x)$ , and  $f(z) = 0.8$ , find  $\sin(2z)$ .
- a) -0.2                      b) 1.64                      c) 0.2                      d) -0.36                      e) NOTA

16) Evaluate the sum: 
$$\sum_{j=0}^{10} \binom{10}{j} 2^j$$

- a) 59049                      b) 2048                      c) 1048576                      d) 2187                      e) NOTA

- 17) If  $A$  is a  $2 \times 2$  matrix, such that  $\det(A) = 8$ , then find the value of  $\det(A^{-1})$ .
- a) 8                      b) 1                      c)  $\frac{1}{8}$                       d)  $\frac{1}{64}$                       e) NOTA
- 18) If  $\cos(x) = 0.5$ , then  $\cos(3x) = ?$
- a) -1                      b) 0                      c) 0.5                      d) 1                      e) NOTA
- 19) In triangle  $ABC$ ,  $AB = 5$ ,  $AC = 3$ , and  $BC = 4$ . Point  $D$  is on side  $AB$ , such that  $AD = 2$ . Determine  $(CD)^2$ .
- a) 6.25                      b) 5.8                      c) 5.76                      d) 2.4                      e) NOTA
- 20) Determine the number of solutions to the equation  $15x + 14y = 1024$ , where  $x$  and  $y$  are positive integers.
- a) Infinite                      b) 5                      c) 7                      d) 10                      e) NOTA
- 21) In Pascal's Triangle, the elements in the first diagonal are all 1's and the elements in the second diagonal are the integers 1, 2, 3, etc. What is the sum of the first twenty elements in the third diagonal?
- a) 1350                      b) 1540                      c) 1840                      d) 2140                      e) NOTA
- 22) A regular tetrahedron has a total surface area of  $144\sqrt{3}$ . Find the ratio of its volume to the area of one of its faces.
- a)  $2 \sec(60^\circ) \sec(45^\circ)$                       b)  $2 \csc(30^\circ) \csc(45^\circ)$   
 c)  $\sec(60^\circ) \sec(30^\circ) \sec(45^\circ)$                       c)  $4 \cot(30^\circ) \cos(45^\circ)$                       e) NOTA
- 23) What is the period of  $f(x) = 8 - 6 | \sin^3(2\pi x) |$ ?
- a)  $\frac{2}{3}$                       b)  $\frac{1}{6}$                       c)  $\frac{1}{2}$                       d)  $\frac{1}{3}$                       e) NOTA

**PRECALCULUS INDIVIDUAL TEST**

**JANUARY 23, 1993**

- 24) Three identical, unfair (six-sided) die are rolled simultaneously. On each, the probability of rolling a 1 is  $\frac{1}{3}$ , a 2 is  $\frac{1}{5}$ , a 3 is  $\frac{1}{6}$ , a 4 is  $\frac{1}{30}$ , and a 5 is  $\frac{2}{15}$ . What is the probability of rolling two 1's and one 6?
- a)  $\frac{1}{45}$                       b)  $\frac{2}{45}$                       c)  $\frac{1}{15}$                       d)  $\frac{4}{15}$                       e) NOTA
- 25) The sides of a triangle are in a geometric sequence. Which of the following could be the common ratio,  $r$ ?
- a) 0.3                      b) 0.5                      c) 1.3                      d) 1.7                      e) NOTA
- 26) Given that  $R = (P \wedge Q') \vee (P \wedge Q)$ . If the values of P and Q (true or false) are chosen at random, what is the probability that R is true?
- a) 0                      b) 0.25                      c) 0.5                      d) 0.75                      e) NOTA
- 27) Let triangle ABC have  $A=60^\circ$ ,  $AB=8$ , and  $BC=7$ .  $AC=?$
- a) 5                      b) 3                      c) 8                      d) 10                      e) NOTA
- 28) What is the maximum number of distinct regions that a rectangle can be divided into by a circle?
- a) 3                      b) 4                      c) 5                      d) 6                      e) NOTA
- 29) A number  $x$  is equal to  $(0.222\dots)_5$  (base 5) and equal to  $(0.333\dots)_n$  (base  $n$ ). Find  $n$ .
- a) 6                      b) 7                      c) 8                      d) 9                      e) NOTA
- 30) If a person was lucky enough to have answered all the questions correct so far and he makes a random decision on this question, what is the probability he will receive a perfect score of 120?
- a)  $\frac{1}{6}$                       b)  $\frac{1}{5}$                       c)  $\frac{1}{3}$                       d)  $\frac{1}{2}$                       e) NOTA