

15

Riverdale

**SOLUTIONS-**  
**PRE - CALC WRITTEN TEST**  
**JANUARY 12, 1991 REGIONAL**

b. #1.

$$\begin{array}{r}
 a^2 - ab - ac + b^2 + c^2 + 2bc \\
 \hline
 a+b+c \left\{ \begin{array}{l} a^3 + b^3 + c^3 \\ -a^3 + a^2b + a^2c \\ \hline -a^2b - a^2c + b^3 + c^3 \\ + a^2b + ab^2 + abc \\ \hline -a^2c + ab^2 + abc + b^3 + c^3 \\ + a^2c + abc + ac^2 \\ \hline -ab^2 + ac^2 + 2abc + b^3 + c^3 \\ -ab^2 + b^3 + bc^2 \\ \hline -ac^2 + 2abc - b^2c + c^3 \\ -ac^2 + c^3 + bc^2 \\ \hline 2abc - b^2c - bc^2 \\ -2abc + 2b^2c + 2bc^2 \\ \hline \boxed{-3b^2c - 3bc^2} \text{ ANS.} \end{array} \right.
 \end{array}$$

d. #2.  $a^{-5} - 5a^{-6}b + 15a^{-7}b^2 - 35a^{-8}b^3 + \boxed{70a^{-9}b^4} - \dots$

d. #3.  $\cos 2x = 1 - 2\left(\frac{3}{4}\right)^2 = -\frac{1}{8}$   
 $\cos 4x = 2\left(-\frac{1}{8}\right)^2 - 1 = \boxed{-\frac{31}{32}}$

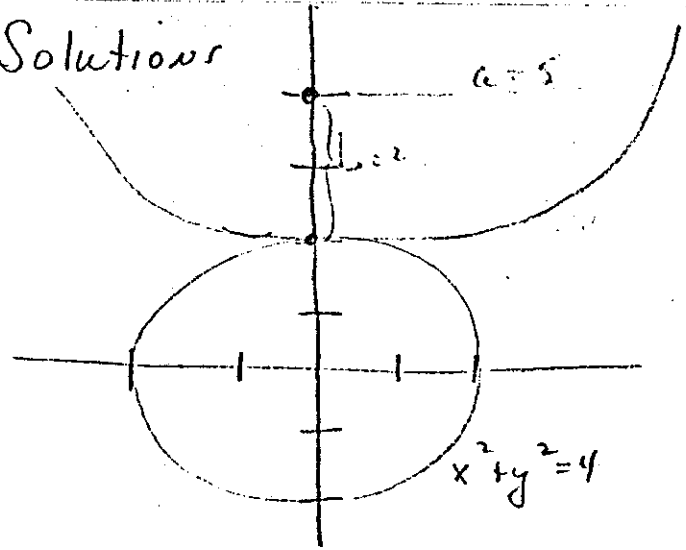
c. #4.  $\frac{6}{36} + \frac{9}{36} = \frac{15}{36} = \boxed{\frac{5}{12}}$

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b. #5.

Pre-calc - Solutions



$$\begin{cases} x^2 + y^2 = 4 \\ 4x^2 + 25y^2 - 200y = -300 \end{cases}$$

$$4x^2 + 25(y^2 - 8y) = -300$$

$$4x^2 + 25(y - 4)^2 = -300 + 400$$

$$\frac{x^2}{25} + \frac{(y-4)^2}{4} = 1 \quad c = (0, 4)$$

a = 5   b = 2

ANS: 1

c. #6

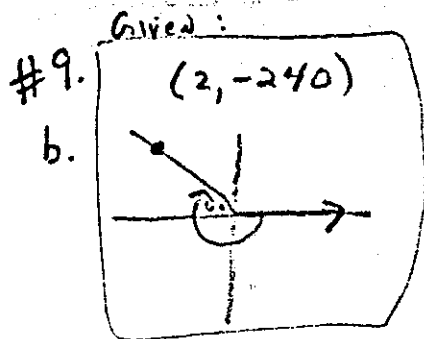
p	q	~p	~q	~p → ~q
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

ANS: c

c #7.  $\cos^{-1}(\cos(-\frac{\pi}{3})) = \cos^{-1}(\frac{1}{2}) = \boxed{\frac{\pi}{3}}$

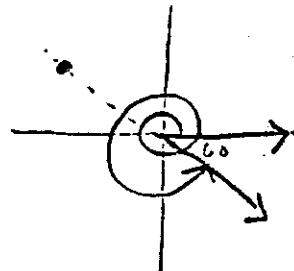
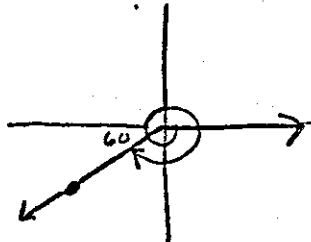
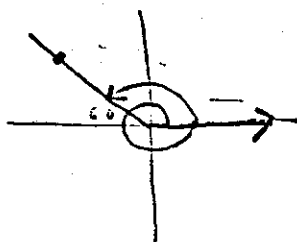
#8  $y = 2 \sin x$  has an amplitude of 2    ANS  $\boxed{e}$

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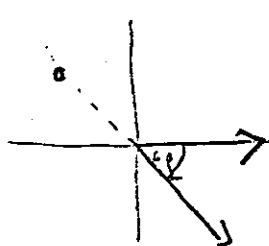


a:

ANS: b



d:



#10 -  $i = 1 \text{ cis } 270 = 1 \text{ cis } 630 = 1 \text{ cis } 990$

b.  $(-i)^{\frac{1}{3}} = 1 \text{ cis } 90 = 1 \text{ cis } 210 = 1 \text{ cis } 330$

#11 c.  $\frac{\cot^2 x + \tan^2 x}{\sin x} = \frac{1}{\sin} \left( \frac{\cos}{\sin} + \frac{\sin^2}{\cos^2} \right) = \frac{\cos}{\sin^2} + \frac{\sin}{\cos^2}$

$$= \frac{\cos^3 + \sin^3}{\sin^2 \cos^2} = \frac{(\cos + \sin)(\cos^2 - \sin \cos + \sin^2)}{\sin^2 \cos^2}$$

$$= \frac{(\cos + \sin)(1 - \sin \cos)}{\sin^2 \cos^2} = \frac{\cos - \sin \cos^2 + \sin - \sin^2 \cos}{\sin^2 \cos^2}$$

$$= \sec^2 \sec - \csc + \csc \sec^2 - \sec$$

$$= \sec^2 \sec - \sec + \csc \sec^2 - \csc$$

$$= \sec(\sec^2 - 1) + \csc(\sec^2 - 1)$$

$$= \boxed{\sec x \cdot \cot^2 x + \csc x \cdot \tan^2 x}$$

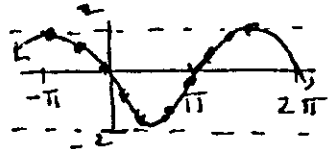
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c. #12.  $-3 \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 1 & 1 \end{vmatrix} + 2 \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & 1 & 0 \end{vmatrix} = -3(-22) + 2(3) = \boxed{72}$

c. #13.  $f(g(2x-1)) = f(4x-3) = \boxed{16x^2 - 24x + 9}$

b. #14.  $\frac{1}{2} \left[ \frac{5!}{5} \right] = \boxed{12}$

d. #15.  $2 \sin(x - \pi) =$    $2 \cos(x + \frac{\pi}{2}) \dots$

a. #16.  $2 \left( \frac{1 + \cos x}{2} \right) - 1$   
 $\boxed{\cos x}$

b. #17.  $\begin{bmatrix} (2 \cdot 3 + 1 \cdot -1) & (2 \cdot 0 + 1 \cdot 2) \\ (3 \cdot 3 + -2 \cdot -1) & (3 \cdot 0 + -2 \cdot 2) \end{bmatrix} = \boxed{\begin{pmatrix} 5 & 2 \\ 11 & -4 \end{pmatrix}}$

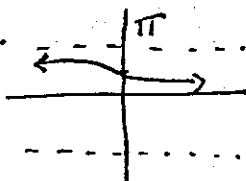
a. #18.  $(2\sqrt{2} \cos \frac{5\pi}{4}) \div (1 \cos \frac{\pi}{2}) = \boxed{2\sqrt{2} \cos(\frac{3\pi}{4})}$

b. #19.  $35^2 = 40^2 + 25^2 - 2(40)(25) \cos \beta$ ,  $\cos \beta = \frac{1}{2}$ ,  $\boxed{\beta = 60^\circ}$

e. #20.  $f = \frac{1}{\text{period}} = \frac{3}{2\pi}$

d. #21.  $\frac{x^2}{9} - \frac{y^2}{3} = 1$

c. #22.  $\frac{n}{2} (a_1 + a_n) = \frac{8}{2} (1 + 15) = \boxed{64}$

c. #23.   $\boxed{(0, \pi)}$

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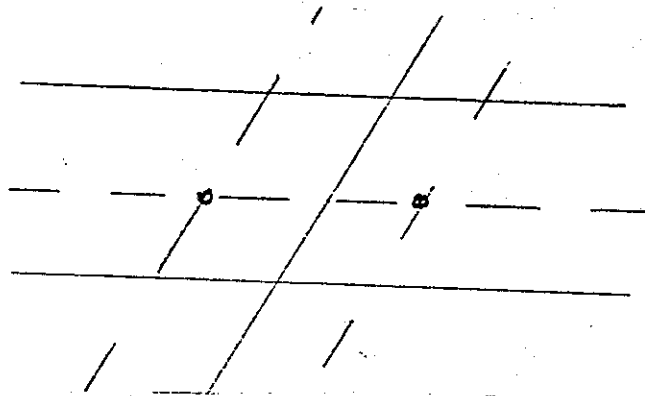
b #24.  $\tan \frac{30}{2} = \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}} = \boxed{2 - \sqrt{3}}$

c #25. polar form of product.

b #26.  $\theta = \frac{10\pi}{1} \cdot \frac{40}{60} = \frac{20\pi}{3}$  coterminal with  $\frac{2\pi}{3}$ .

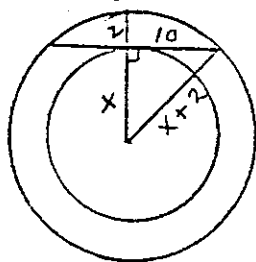
$P = (6 \cos \frac{2\pi}{3}, 6 \sin \frac{2\pi}{3}) = \boxed{(-3, 3\sqrt{3})}$

c #27.



ANS: 2

a #28



$$\begin{aligned} x^2 + 100 &= (x+2)^2 \\ x^2 + 100 &= x^2 + 4x + 4 \\ 4x &= 96 \\ \boxed{x} &= \boxed{24} \end{aligned}$$

d. #29.  $\tan 2x (\tan^2 2x - 1) = 0$

$\tan 2x = 0$        $\tan 2x = \pm 1$

$2x = 0, \pi, 2\pi, 3\pi, \dots, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \dots$

$X = (0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}) + (\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}) = \boxed{11\pi}$

a. #30.  ${}^8C_3 \cdot {}^{10}C_2 = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} \cdot \frac{10 \cdot 9}{2 \cdot 1} = \boxed{9 \cdot 8 \cdot 7 \cdot 5}$