Solutions to Calculus Individual

Lobo - Jan 99

1. $3 + 2x > 0$

$2x > -3$

$x > -\frac{3}{2}$

$\boxed{\left(-\frac{3}{2}, \infty\right)}$

2. $f(x+1) - f(x)$

$|3(x+1)+1| - 5 - \left[|3x+1| - 5\right]$

$|3x+3+1| - 5 - |3x+1| + 5$

$\boxed{|3x+4| - |3x+1|}$

3. $\lim_{x \to -1} x + 3 = 2$

$\lim_{x \to -1} 2x - c = 2$

$\Rightarrow 2(-1) - c = 2$

$-2 - c = 2$

$\boxed{-4 = c}$

4. $\lim_{h \to 0} \frac{\sqrt{(x+h)+2} - \sqrt{x+2}}{h}$

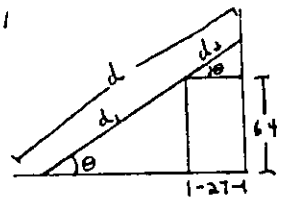
$\frac{\sqrt{(x+h)+2} - \sqrt{x+2}}{h} \cdot \frac{\sqrt{(x+h)+2} + \sqrt{x+2}}{\sqrt{(x+h)+2} + \sqrt{x+2}}$

$\lim_{h \to 0} \frac{x + h + 2 - x - 2}{h\left(\sqrt{(x+h)+2} + \sqrt{x+2}\right)}$

$\lim_{h \to 0} \frac{1}{\sqrt{(x+h)+2} + \sqrt{x+2}}$

$\Rightarrow \boxed{\frac{1}{2\sqrt{x+2}}}$

5. $d = d_1 + d_2$



$d = 64\csc\theta + 27\sec\theta$

$\frac{d}{d\theta}(d) = -64\csc\theta\cot\theta + 27\sec\theta\tan\theta = 0$

Using $\csc\theta = \frac{1}{\sin\theta}$; $\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$; $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\frac{64\cos\theta}{\sin^2\theta} = \frac{27\sin\theta}{\cos^2\theta}$

$64\cos^3\theta = 27\sin^3\theta$

$\tan^3\theta = \frac{64}{27}$

$\tan\theta = \frac{4}{3}$

$\boxed{\theta \approx 53^\circ}$

6. $f(x) = x^{3/2} - 4x^{1/2}$

$\boxed{f'(x) = \frac{3}{2}x^{1/2} - 2x^{-1/2}}$

7. $g(x) = -5f(x)$

$g'(x) = -5f'(x) + 0 \cdot f(x)$

$g'(x) = -5f'(x)$

$g'(-7) = -5 \cdot f'(-7) \Rightarrow -5 \cdot 6 = \boxed{-30}$

8. $f'(x) = 3x^2$

$f'(x) = 3 \Rightarrow 3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$\boxed{x = 1 \Rightarrow y = -1}$

$\boxed{x = -1 \Rightarrow y = -3}$

9. $\int_{-5}^{0} (x^3+x^2-12x)-(-x^2+3x)\,dx +$

$\int_{0}^{3} (-x^2+3x)-(x^3+x^2-12x)\,dx$

$\int_{-5}^{0} (x^3+2x^2-15x)\,dx + \int_{0}^{3} -x^3-2x^2+15x\,dx$

$\frac{1}{4}x^4+\frac{2}{3}x^3-\frac{15}{2}x^2\Big]_{-5}^{0} + \left(-\frac{1}{4}x^4-\frac{2}{3}x^3+\frac{15}{2}x^2\right)\Big]_{0}^{3}$

$\frac{1375}{12} + \frac{117}{4} = \boxed{\frac{863}{6}}$

10. $\frac{2}{3}x^3+\frac{2}{3}(x+5)^{3/2}\Big]_a^b$

$\frac{2}{3}b^3+\frac{2}{3}(b+5)^{3/2}-\frac{2}{3}a^3-\frac{2}{3}(a+5)^{3/2}$

$\boxed{\frac{2}{3}(b^3-a^3)+\frac{2}{3}(b+5)^{3/2}-\frac{2}{3}(a+5)^{3/2}}$

11. $\frac{dy}{dx} = \frac{(x-3)-(x+2)}{(x-3)^2}$

$= \frac{-5}{(x-3)^2}$

$\frac{d^2y}{dx^2} = \frac{0-(-5)(2)(x-3)}{(x-3)^4}$

$= \frac{10(x-3)}{(x-3)^4}$

$\boxed{= \frac{10}{(x-3)^3}}$

12. $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$dV = 4\pi r^2\,dr$

$dV = 4\pi(3.0)^2(.01)$

$\boxed{dV = \pm .36\pi}$

13. $\int x\sqrt{x+1}\,dx$ $\quad u = x+1$

$\int (u-1)u^{1/2}\,du$ $\quad x = u-1$

$\int (u^{3/2}-u^{1/2})\,du$ $\quad du = dx$

$\frac{2}{5}u^{5/2}-\frac{2}{3}u^{3/2}+C$

$\frac{6}{15}(x+1)^{5/2}-\frac{10}{15}(x+1)^{3/2}+C$

$\frac{2}{15}(x+1)^{3/2}\left[3(x+1)-5\right]+C$

$\boxed{\frac{2}{15}(x+1)^{3/2}(3x-2)+C}$

14. $\frac{1}{2-0}\int_0^2 (3x^2-2)\,dx$

$\frac{1}{2}\left[x^3-2x\right]_0^2$

$\frac{1}{2}(8-4) = \boxed{2}$

15. $P_u = 225e^{0.007t}$

$P_M = 62e^{0.024t}$

$225e^{0.007t} = 62e^{0.024t}$

$3.6290 = e^{0.017t}$

$\ln 3.6290 = 0.017t$

$t \approx 75.82$

$75 + 1980 = \boxed{2055}$

16. $14x+6y+6xy'+18yy' = 0$

$y'(6x+18y) = -14x-6y$

$\boxed{y' = -\frac{14x+6y}{6x+18y}}$

17. $f'(x) = -2x+12$

$(-\infty, 6)$	$(6, \infty)$
$f'(0) > 0$	$f'(7) < 0$
Increasing	Decreasing

18. $x = \tan(x+y)$

$1 = (1+y')\sec^2(x+y)$

$1 - \sec^2(x+y) = y'\sec^2(x+y)$

$\frac{-\tan^2(x+y)}{\sec^2(x+y)} = y'$

$-\frac{\sin^2(x+y)}{\cos^2(x+y)} \cdot \cos^2(x+y) = y'$

$\boxed{-\sin^2(x+y) = y'}$

19. $\int \left(8 + \frac{9x}{x^2+1}\right) dx$

$\int 8\,dx + 9\int \frac{x}{x^2+1}\,dx$

$\boxed{8x + \frac{9}{2}\ln|x^2+1| + C}$

20. $\int \frac{1}{x^2 e^{3/x}}\,dx \qquad u = -\frac{3}{x}$

$du = \frac{3}{x^2}\,dx$

$\frac{1}{3}\int e^u\,du$

$\frac{1}{3}e^u + C \Rightarrow \boxed{\frac{1}{3}e^{-3/x} + C}$

21. $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sin^2 2\theta\,d\theta$

$\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{1-\cos 4\theta}{2}\,d\theta$

$\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta \Big]_{\frac{\pi}{8}}^{\frac{\pi}{4}}$

$\left(\frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{8}\sin\pi\right) - \left(\frac{1}{2}\cdot\frac{\pi}{8} - \frac{1}{8}\sin\frac{\pi}{2}\right)$

$\frac{\pi}{8} - \frac{\pi}{16} + \frac{1}{8}$

$\frac{2\pi - \pi + 2}{16}$

$\boxed{\frac{\pi+2}{16}}$

22. $\lim_{x\to 0} \frac{e^{x^2}-1}{4x}$

$\lim_{x\to 0} \frac{e^{x^2}}{2} = \boxed{\frac{1}{2}}$

23. $\frac{dy}{dx} = Ky$

$y = Ce^{Kt}$

$y = y_0 e^{Kt}$

$\frac{1}{2}y_0 = y_0 e^{40K}$

$\ln\frac{1}{2} = 40K$

$K = \frac{\ln\frac{1}{2}}{40}$

$K \approx -.01733$

at $t = 48 \Rightarrow y = y_0 e^{\left(\frac{\ln\frac{1}{2}}{40}\right)48}$

$y = y_0 (0.4353)$

approx. $\boxed{43.53\%}$

24. $\int \frac{\sec^3\theta\tan\theta}{\sec^2\theta}\,d\theta$

$\Rightarrow \int \sec\theta\tan\theta\,d\theta$

$\Rightarrow \boxed{\sec\theta + C}$

25. $v(t) = 6t - 2$

$v(2) = 6(2) - 2$

$= 12 - 2$

$\boxed{= 10}$

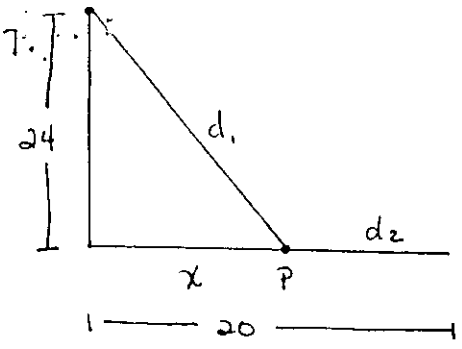
26. $xe^y + 1 = xy$

$e^y + xy'e^y = y + xy'$

$y'(xe^y - x) = y - e^y$

$\boxed{y' = \frac{y - e^y}{xe^y - x}}$

27.



$d_1 = (24^2 + x^2)^{1/2}$ $\quad d_2 = 20 - x$

$d = r \cdot t$ $\quad t_2 = \frac{20 - x}{13}$

$t_1 = \frac{(24^2 + x^2)^{1/2}}{5}$

$T = t_1 + t_2 \Rightarrow T = \frac{(24^2 + x^2)^{1/2}}{5} + \frac{20 - x}{13}$

$T' = \frac{1}{5} x (24^2 + x^2)^{-1/2} - \frac{1}{13} = 0$

$13x = 5(24^2 + x^2)^{1/2}$

$169 x^2 = 25 (24^2 + x^2)$

$x^2 = \frac{(25)(24^2)}{144}$

$\boxed{x = 10 \text{ miles}}$

28. $y' = 1 + \sin x = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$

at $x = 0$, $y = -1$

at $x = \frac{3\pi}{2}$, $y = \frac{3\pi}{2}$

at $x = 2\pi$, $y = 2\pi - 1$

29. $3^x \cdot 3x^2 + x^3 (\ln 3) 3^x$

$\boxed{3^x x^2 (3 + x \ln 3)}$

30. $f'(x) = -3x^2 + 6x$

$-3x(x - 2) = 0$

$x = 0, 2$

$x = 0, y = -2$

$x = 2, y = 2$

$\boxed{(0, -2) \quad (2, 2)}$