CALCULUS INDIVIDUAL TEST – SOLUTIONS

River Ridge

1. $\lim\limits_{x \to -2} \dfrac{(x+2)(x+2)}{(x+2)(x+1)} \Rightarrow \lim\limits_{x \to -2} \dfrac{x+2}{x+1} = \boxed{0}$

E 2. $x^2 - 16 = 0 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

Checking regions $\Rightarrow$ $x < -4$ and $x > 4$

3. $x + 1 < 3x - 1$ OR $x + 1 < -3x + 1$

$-2x < -2$ $\qquad 4x < 0$

$x > 1$ $\qquad x < 0$

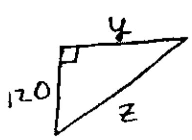
4. $\ln y = \frac{1}{3}\ln(x^2+1) + \frac{1}{3}\ln(x-1) - 2\ln(2x+1) - 4\ln(3x-2)$

$$\frac{y'}{y} = \frac{1}{3}\left(\frac{2x}{x^2+1}\right) + \frac{1}{3}\left(\frac{1}{x-1}\right) - 2\left(\frac{2}{2x+1}\right) - 4\left(\frac{3}{3x-2}\right)$$

$$\frac{y'}{y} = \frac{2x}{3(x^2+1)} + \frac{1}{3(x-1)} - \frac{4}{2x+1} - \frac{12}{3x-2}$$

$$y' = \frac{\sqrt[3]{(x^2+1)(x-1)}}{(2x+1)^2(3x-2)^4}\left[\frac{2x}{3(x^2+1)} + \frac{1}{3(x-1)} - \frac{4}{2x+1} - \frac{12}{3x-2}\right]$$

5. $\lim\limits_{x \to \infty} \dfrac{2x^2 - 6x + 1}{1 + x^2} \Rightarrow \lim\limits_{x \to \infty} \dfrac{2 - \frac{6}{x} + \frac{1}{x^2}}{\frac{1}{x^2} + 1} = 2$ $\quad \boxed{\therefore y = 2}$

6. 

$\frac{dy}{dz} = 10$ ft./s

when $z = 150$

$y = 90$

$120^2 + y^2 = z^2$

$\cancel{2}y\frac{dy}{dt} = \cancel{2}z\frac{dz}{dt}$

$\frac{y}{z}\frac{dy}{dt} = \frac{dz}{dt}$

$\frac{90}{150}(10) = \frac{dz}{dt}$

$\boxed{6 \text{ ft/s} = \frac{dz}{dt}}$

7. $7x^2 + 6xy + 9y^2 = 0$

$14x + 6y + 6x\frac{dy}{dx} + 18y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(6x + 18y) = -14x - 6y$

$$\boxed{\frac{dy}{dx} = -\frac{14x+6y}{6x+18y}}$$

8. $\int (2x^2-5)\,dx$

$y = \frac{2}{3}x^3 - 5x + C$

$2 = \frac{2}{3}(3)^3 - 5(3) + C$

$-1 = C$

$\boxed{y = \frac{2}{3}x^3 - 5x - 1}$

9. $\int_0^3 |3x-4|\,dx \Rightarrow \int_0^{4/3} -(3x-4)\,dx + \int_{4/3}^3 (3x-4)\,dx$

$= \left(-\frac{3}{2}x^2 + 4x\right)\Big]_0^{4/3} + \left(\frac{3}{2}x^2 - 4x\right)\Big]_{4/3}^3$

$= \left(-\frac{8}{3} + \frac{16}{3}\right) + \left[\left(\frac{27}{2} - 12\right) - \left(\frac{8}{3} - \frac{16}{3}\right)\right]$

$\boxed{= \frac{41}{6}}$

10. $h(x) = \frac{f(x)}{g(x)} \Rightarrow h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$= \frac{1 \cdot 14 - 0 \cdot \frac{1}{7}}{1^2} \Rightarrow \boxed{14}$

11. $y = Ce^{kt} \Rightarrow 1000 = 500e^{2k} \Rightarrow 2 = e^{2k}$

$C = 500,\ t = 5 \Rightarrow y = 500e^{5k}$

$y = 500(e^{2k})^{5/2} \Rightarrow y = 500(2)^{5/2} \approx \boxed{2828}$

12. $s = -\frac{35}{2}t^2 + 58t + 91 \Rightarrow v = -35t + 58$

at $t = 1 \Rightarrow v = -35 + 58 \Rightarrow 23$

13. $12 = \sqrt{(6-x)^2 + (-1-9)^2} \Rightarrow 144 = 36 - 12x + x^2 + 100$

$144 = 136 - 12x + x^2$

$x^2 - 12x - 8 = 0$

$x = \frac{12 \pm 4\sqrt{11}}{2} \Rightarrow \boxed{6 \pm 2\sqrt{11}}$

14. $2x + 3y + 3xy' + 2yy' = 0 \Rightarrow y' = \frac{-2x-3y}{3x+2y}$

at $(1,1) \Rightarrow y' = -1$

slope of normal $= 1$

$y - 1 = 1(x-1)$

$y - 1 = x - 1$

$\boxed{x - y = 0}$

5. $\cancel{2}x + \cancel{2}yy' = \cancel{2}y + \cancel{2}xy'$

$yy' - xy' = y - x$

$y'(y-x) = y - x$

$\boxed{y' = 1}$

16. $v = \frac{1}{2}(t^3+1)^{-1/2}(3t^2) \Rightarrow \frac{3}{2}t^2(t^3+1)^{-1/2}$

$a = \frac{3}{2}(2t)(t^3+1)^{-1/2} + \frac{3}{2}t^2\left(-\frac{1}{2}\right)(3t^2)(t^3+1)^{-3/2}$

$= 3t(t^3+1)^{-1/2} - \frac{9t^4}{4}(t^3+1)^{-3/2}$

at 2 s $\Rightarrow 3(2)(2^3+1)^{-1/2} - \frac{9(2)^4}{4}(2^3+1)^{-3/2} \Rightarrow 2 - \frac{36}{27} \Rightarrow \boxed{\frac{2}{3}}$

17. $A = Pe^{rt} \Rightarrow 3 = e^{21r} \Rightarrow \ln 3 = 21r \Rightarrow r = \frac{\ln 3}{21} \approx \boxed{5.23\%}$

18. $\int x^2(x^3+5)^6\,dx$ $\qquad u = x^3+5$, $du = 3x^2\,dx$

$= \frac{1}{3}\int u^6\,du$

$= \frac{1}{3}\left(\frac{u^7}{7}\right) + C \Rightarrow \boxed{\frac{1}{21}(x^3+5)^7 + C}$

19. $f'(x) = (x-4)^2 + 2(x-4)(x+3) \Rightarrow (x-4)(3x+2)$

$f''(x) = (3x+2) + (x-4)$

$0 = 4x - 2$

$\boxed{x = \frac{1}{2} \Rightarrow \text{possible point of inflection}}$

20. $\sin(x+y) = 1 \Rightarrow (1+y')\cos(x+y) = 0$

$\Rightarrow \cos(x+y) + y'\cos(x+y) = 0$

$\boxed{y' = -1}$

21. $\sin x + \cos x = y$

$\cos x - \sin x = y' \Rightarrow \cos x - \sin x = 0$

$\cos x = \sin x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$\sin\frac{\pi}{4} + \cos\frac{\pi}{4} = \sqrt{2}$

$\sin\frac{5\pi}{4} + \cos\frac{5\pi}{4} = -\sqrt{2}$

$\boxed{\left(\frac{\pi}{4}, \sqrt{2}\right) \text{ and } \left(\frac{5\pi}{4}, -\sqrt{2}\right)}$

22. $y = \frac{1}{2}\left[\ln(x+2) - \ln(x^2-4)\right]$

$y' = \frac{1}{2}\left(\frac{1}{x+2} - \frac{2x}{x^2-4}\right)$

$= \frac{1}{2}\left(\frac{x-2-2x}{x^2-4}\right) \Rightarrow \frac{1}{2}\left(\frac{-x-2}{x^2-4}\right) \Rightarrow \frac{1}{2}\left(-\frac{\cancel{x+2}}{(\cancel{x+2})(x-2)}\right)$

$\boxed{= -\frac{1}{2(x-2)}}$

23. $\lim_{x\to 0} \frac{x}{\tan x} \Rightarrow \lim_{x\to 0} \frac{1}{\sec^2 x} = \boxed{1}$

24. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ $\qquad f'(x) = 3x^2 - 2$

x_n	$f(x_n)$	$f'(x_n)$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1.5000	-1.625	4.75	1.8421
1.8421	.5666	8.1800	1.7728
1.7728	.0260	7.4285	$\boxed{1.7693}$

25. $\int_0^K (2Kx - x^2)\,dx = Kx^2 - \frac{1}{3}x^3\Big]_0^K = 144$

$K^3 - \frac{1}{3}K^3 = 144$

$3K^3 - K^3 = 432$

$2K^3 = 432$

$K^3 = 216$

$\boxed{K = 6}$

26. $\int_1^2 (x-1) - (2x^2-5x+3)\,dx \Rightarrow \int_1^2 (-2x^2+6x-4)\,dx$

$\Rightarrow -\frac{2}{3}x^3 + 3x^2 - 4x \Rightarrow \left(-\frac{16}{3} + 12 - 8\right) - \left(-\frac{2}{3} + 3 - 4\right) = \boxed{\frac{1}{3}}$

27. $\lim_{\Delta x \to 0} \frac{[2(x+\Delta x)^2 + 4] - (2x^2+4)}{\Delta x}$

28. $x^2+3x \overline{\big)\, x^4 + 3x^3 - 2x - 3}$ $\Rightarrow \int \left(x^2 - \frac{2x+3}{x^2+3x}\right) dx$

$\Rightarrow \boxed{\frac{1}{3}x^3 - \ln|x^2+3x| + C}$

29. $y = \ln xe^x \Rightarrow y' = \frac{e^x + xe^x}{xe^x} \Rightarrow y' = \frac{1}{x} + 1$

at $x = 3 \Rightarrow y' = \frac{1}{3} + 1 = \boxed{\frac{4}{3}}$

30. $\frac{dy}{dx} = 2\sin x \cos x \Rightarrow \sin 2x$

$\frac{d^2y}{dx^2} = \boxed{2\cos 2x}$