ANSWER KEY

January MAØ Regional 1997

Multiple Choice Answers

1) D
2) A
3) A
4) B
5) B
6) D
7) B
8) C
9) A
10) D
11) D
12) A
13) D
14) E
15) D
16) E
17) A
18) C
19) C
20) C
21) B
22) B
23) C
24) C
25) D
26) C
27) B
28) A
29) B
30) A

Team Round Answers

1) Answer: $9\sqrt{2}$

2) Answer: 9

3) Answer: 3/2

4) Answer: 30.396 ft^3

5) Answer: $1+\dfrac{1}{1-\frac{1}{\pi}}$ *or* $1+\dfrac{\pi}{\pi-1}$ *or* $\dfrac{2\pi-1}{\pi-1}$

6) Answer: $y-1=\frac{1}{2}(x-1)$

7) Answer: $w\in\left(2-\sqrt{3},\ 2+\sqrt{3}\right)$

8) Answer: $y=e$

9) Answer: 42

10) Answer: $a=2,\ b=6$

11) Answer: -7/2

12) Answer: -4

13) Answer: $2e^4$

14) Answer: 0

15) Answer: -5.21

/12

Calculus Individual Test SOLUTION GUIDE January MAØ Regional 1997

1) D. The curve is the graph of $3x^2+6x$ which is the derivative of $f(x) = x^3+3x^2-4$.

2) A. $\lim_{x\to\frac{\pi}{4}} \dfrac{\sin^2 x - \sin^2\frac{\pi}{4}}{x-\frac{\pi}{4}} = D_x(\sin^2 x)\Big|_{\frac{\pi}{4}} = 2\sin\frac{\pi}{4}\cos\frac{\pi}{4} = 1$

3) A. The function is undefined at -2 and 2, and between -1 and 1.

4) B. $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)},\ x_1 = 1 - \dfrac{-1}{2} = \dfrac{3}{2}$

$x_2 = \dfrac{3}{2} - \dfrac{1/4}{3} = \dfrac{17}{12} \approx 1.417$

5) B. $3y^2-6y = 2\dfrac{dx}{dy},\ \dfrac{dx}{dy} = 0 = 3y^2-6y$

$y=2,\ x=-2$

6) D. $h'(x) = f'(x)\,g(x) + f(x)\,g'(x)$ so $g'(x) = \dfrac{h'(x) - f'(x)\,g(x)}{f(x)}$. $g'(2) = 1/2$.

7) B. $\pm\sqrt{\lim_{x\to 2^+}\left(\dfrac{x-2}{\sqrt{x^2-2x}}\right)^2} = \pm\sqrt{\lim_{x\to 2^+}\dfrac{x^2-4x+4}{x^2-2x}} = \pm\sqrt{\lim_{x\to 2^+}\dfrac{2x-4}{2x-2}} = 0.$

8) C. at $x=-1$ and 1, $\frac{1}{2}x^2+\frac{1}{2} = |x|$

and $D_x\left(\frac{1}{2}x^2+\frac{1}{2}\right) = D_x(|x|)$

9) A. Definition of derivative of $\sqrt[3]{x}$. Does not exist at x=0.

10) D. Graph is concave down everywhere but undefined at 0.

11) D.

$D_x[f(g(x))] = f'(g(x))g'(x) = -f'(-g(x))\,g'(-x)$
$= -f'(g(-x))g'(-x)$, odd.

12) A.

$\lim_{h\to 0}\dfrac{4-\sqrt{h+16}}{4h} = \lim_{h\to 0}\dfrac{4-\sqrt{h+16}}{4h}\left(\dfrac{4+\sqrt{h+16}}{4+\sqrt{h+16}}\right)$

$= \lim_{h\to 0}\dfrac{16-(h+16)}{4h\left(4+\sqrt{h+16}\right)} = \lim_{h\to 0}\dfrac{-1}{4\left(4+\sqrt{h+16}\right)} = -1/32$

13) D. $f'(x) = \frac{1}{2\sqrt{x}} = \frac{3-1}{9-1}, x = 4$

14) E. $f'(x) = 3x^2 + 4x + 1$ which has zeros at -1 and -1/3.
$f(-1) = 3,\ f(-1/3) = 77/27$ the local max and min. But $f(-2) = 1$.
NOTA.

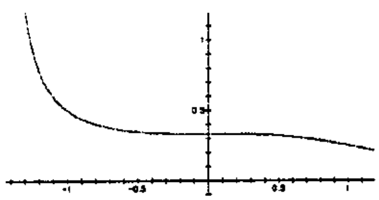
15) D. $h'(x) = f'(g(x))g'(x)$ therefore $\frac{h'(1)}{g'(1)} = f'(g(1)) = f'(2) = 5.$

16) E. It cannot be made continuous by using a value at f(1) because it has a vertical asymptote there.

17) A. Initial time is t-10 seconds. At this time, initial velocity is 5 ft/s and position is 150 ft. $s = 20(t-10)^2 + 5(t-10) + 150$

18) C. (2,2) is on the curve. So, $\frac{dy}{dx} = \frac{1}{y} = \frac{1}{2},\ (y-2) = \frac{1}{2}(x-2),\ y = \frac{1}{2}x + 1.$

19) C. Graph suggests function is continuous between 0 and -1, analytically discontinuous at $-\sqrt[3]{3}$, so the fundamental theorem applies. $\frac{1}{-1^3+3} = \frac{1}{2}$.



20) C.
$\frac{dv}{dt} = 800,\ V = \frac{\pi r^2 h}{3},\ r = h/2$ so $V = \frac{\pi h^3}{12}$.
$\frac{dv}{dt} = 800 = \frac{\pi h^2}{4}\frac{dh}{dt},\ h = 20$ so $\frac{dh}{dt} = \frac{8}{\pi}$

21) B. $x^2 + y^2 + 2x + 2y = 0$ is the circle of radius $\sqrt{2}$ centered at (-1,-1). A normal line must go through the center. 2(-1) - (-1) = c = -1.

22) B.
$x^4 + x^2y^2 - y^2 = 0$
$4x^3 + x^2 2y\,y' + 2x\,y^2 - 2y\,y' = 0$
$2y(x^2-1)\,y' = -2x(2x^2+y^2)$
$y' = \frac{x(2x^2+y^2)}{y(1-x^2)}$

$y' = \frac{\frac{1}{2}\left(\frac{1}{2}+\frac{1}{12}\right)}{\frac{1}{2\sqrt{3}}\left(1-\frac{1}{4}\right)} = \frac{\frac{7}{24}}{\frac{1}{2\sqrt{3}} - \frac{1}{8\sqrt{3}}} = \frac{7\sqrt{3}}{9}$

23) C. $\int_0^1 2\pi(x)(1-x^2)dx = 2\pi\left(\frac{x^2}{2}-\frac{x^4}{4}\right)\Big|_0^1 = \frac{\pi}{2}$

24) C. $f'(x) = 3x^2+6x+6$, so $f''(x) = 6x+6$, $f''(-1) = 0$ therefore (-1, -1)

25) D. $f^{-1\prime}(11) = \frac{1}{f'(f^{-1}(11))} = \frac{1}{f'(2)} = \frac{1}{13}$

26) C.

$\cot(x-y) = -x$

$-\csc^2(x-y)\left(1-\frac{dy}{dx}\right) = -1$

$-\csc^2(x-y)+\csc^2(x-y)\frac{dy}{dx} = -1$

$\csc^2(x-y)\frac{dy}{dx} = -1+\csc^2(x-y)$

$\frac{dy}{dx} = -\sin^2(x-y)+1 = \cos^2(x-y)$

27) B. When $x = 2$ $y = 3$. $\frac{dy}{dx} = \frac{-2}{x^2-2x+1}$ which at 2 equals -2.

$y-3 = -2(x-2)$ so $y = -2x+7$.

28) A. This curve has zeros at x = -3 and x = 1, and it is concave down.

$\int_{-3}^{1} -x^2-2x+3\ dx + \int_{2}^{1} -x^2-2x+3\ dx$

$\frac{-x^3}{3}-x^2+3x\Big|_{-3}^{1} = \frac{32}{3}$, $\frac{-x^3}{3}-x^2+3x\Big|_{2}^{1} = \frac{7}{3}$, answer = 13.

29) B. $f''(x) = 6x^3-12x^2+6x = (x)(x-1)^2$ which changes sign only at (0,1).

30) A. $h(x) = x+1$, $h'(x) = 1$, $x \neq 1$ Undefined.