4

Calculus Individual Solutions
January Regional

① $f(x)=\sqrt{\frac{x}{3-x}}$

Domain: $x \neq 3$

$- - - - - \; + + + + + +$

$+ + + + + + 0 + + + + + \circ - - - -$ (3)

$[0,3)$ Ⓑ

② $y=(x-1)^2 e^x$

$y'=(x-1)^2 e^x + e^x \cdot 2(x-1)$

$y'=e^x(x^2-1)$ Ⓑ

③ $\lim_{x\to-\infty} \frac{\pi^x-\pi^{-x}}{\pi^x+\pi^{-x}} = \lim_{x\to-\infty} \frac{\pi^{2x}-1}{\pi^{2x}+1} = -1$

Ⓐ

④ $y=3x^4-10x^3-12x^2+12x-7$

$y'=12x^3-30x^2-24x+12$

$y''=36x^2-60x-24=12(3x+1)(x-2)$

y'' $+ + + + + \; - - - - - - - \; + + + + +$ ($-\frac{1}{3}$, 2)

$(-\frac{1}{3},2)$ Ⓐ

⑤ $y=5-x^2$ $\quad \frac{y-0}{x-3}=\frac{5-x^2}{x-3}$

$y'=-2x$

and $\frac{5-x^2}{x-3}=-2x$

$x^2-6x+5=0 \Rightarrow \cancel{x=5}, x=1$

$x=1, y=4;\ d=\sqrt{(1-3)^2+(4-0)^2}$

$=2\sqrt{5}$ Ⓓ

⑥ e representation

I, II, IV Ⓒ

⑦ $x^2-xy+y^2=3$

$2x-xy'-y+2yy'=0$ Ⓓ

$y'=\frac{2x-y}{x-2y}$

⑧ $\lim_{x\to2} \frac{4-x^2}{3-\sqrt{x^2+5}}$

L'Hopital's Rule $\rightarrow \lim_{x\to2} \frac{-2x}{-\frac{1}{2}(x^2+5)^{-1/2}\cdot 2x}$

$=\lim_{x\to2} 2\sqrt{x^2+5} = 6$ Ⓐ

⑨ $y=(\tan x \sin 2x)/2$

$y'=\frac{1}{2}\left[2\tan x \cos 2x + \sin 2x \sec^2 x\right]$

$=\sin x\cos x - \sin^2 x \tan x + \tan x$

$=\tan x \cos^2 x + \sin x \cos x$

$=2\sin x \cos x = \sin 2x$ Ⓒ

⑩ $F(x)=\begin{cases} x+1, & 1<x<3 \\ x^2+bx+c, & |x-2|\geq 1 \end{cases}$

$\lim_{x\to1} x^2+bx+c = \lim_{x\to1} x+1 = 2$

$1+b+c=2 \Rightarrow b+c=1$

$\lim_{x\to3} x^2+bx+c = \lim_{x\to3} x+1$

$9+3b+c=4$

$3b+c=-5$

$b=-3,\ c=4$ Ⓑ

⑪ $f(x)=e^{-x}\ln x$

$f'(x)=e^{-x}\cdot\frac{1}{x}+(\ln x)e^{-x}\cdot -1$

$f'(1)=\frac{1}{e}$ Ⓔ

⑫ $x^2-y^2=16$ hyperbola

$2x-2yy'=0 \Rightarrow y'=\frac{x}{y}$

$y'=\frac{x}{\sqrt{x^2-16}}$ $\quad (2,-2)$

$\frac{\sqrt{x^2-16}+2}{x-2}=\frac{x}{\sqrt{x^2-16}}$

$x^2-16+2\sqrt{x^2-16}=x^2-2x$

$\sqrt{x^2-16}=(8-x)^2$

$16x=80 \Rightarrow x=5$

$\therefore y'=\frac{5}{3}$ & $5x-3y=16$ Ⓐ

⑬ $y=\arcsin x-\sqrt{1-x^2}$

$y'=\frac{1}{\sqrt{1-x^2}}-\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\cdot -2x$

$y'=\frac{x+1}{\sqrt{1-x^2}}$ Ⓒ

⑭ $g(x)=x^{4/3}-x^{1/3}$

$g'(x)=\frac{4}{3}x^{1/3}-\frac{1}{3}x^{-2/3}$ $\quad (-\infty,\frac{1}{4}]$

$\frac{4x^{1/3}}{3}-\frac{1}{3x^{2/3}}=0$ $\quad$ Ⓒ or E

$4x-1=0 \Rightarrow x=\frac{1}{4}$

$g'(x)$ $- - - - - \; - - - \; + + + + + +$ (0, $\frac{1}{4}$)

⑮ $f(x)=ax^4+bx^2,\ ab>0$

no inflection pts Ⓓ

⑯ $pV^{1.4}=c$

$p\cdot 1.4V^{.4}\frac{dV}{dt}+V^{1.4}\frac{dp}{dt}=0$

$(40)(1.4)(60)^{.4}\frac{dV}{dt}+(60)^{1.4}(3)=0$

$\frac{dV}{dt}=-3.2\ \text{cm}^3/\text{sec}$ Ⓑ

⑰ $f(t)=t-\tan t$

$f'(t)=1-\sec^2 t$

$1-\frac{1}{\cos^2 t}=0 \Rightarrow \cos^2 t=1$

$\cos t=\pm 1$ Ⓐ

$f(0)=0$

$f(\frac{\pi}{4})=\frac{\pi-4}{4}$ $\quad f(-\frac{\pi}{4})=\frac{4-\pi}{4}$ max

⑱ $\lim_{x\to3}(2+5x)=17$

$0<|x-3|<\delta \Rightarrow |2+5x-17|<\epsilon$

$|5x-15|<\epsilon$

$5|x-3|<\epsilon \Rightarrow |x-3|<\frac{\epsilon}{5}$

Choose $\delta=\epsilon/5$ Ⓒ

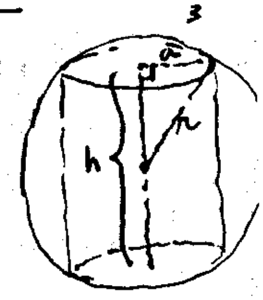
⑲ $V_s=\frac{4}{3}\pi r^3;\ V_c=h\pi a^2$ $\quad V=h\pi\left(r^2-\frac{h^2}{4}\right)$

$\frac{V_s}{V_c}=\frac{\frac{4}{3}\pi r^3}{\pi\left[\frac{2r\sqrt{3}}{3}\right]\left[r^2-\frac{12r^2}{9\cdot4}\right]}$ $\quad V=h\pi r^2-h^3\frac{\pi}{4}$

$V'=\pi r^2-\frac{3\pi h^2}{4}=0$

$=\frac{\frac{4}{3}}{\left(\frac{2\sqrt{3}}{3}\right)\left(\frac{2}{3}\right)}$ $\quad h=\frac{2r\sqrt{3}}{3}$

$=\frac{\sqrt{3}}{1}$ Ⓓ



$\left(\frac{h}{2}\right)^2+a^2=r^2$

$a^2=r^2-\frac{h^2}{4}$

(20) $y = 6\cdot 3^{\cos x}$

$y' = 6\cdot(3^{\cos x})(\ln 3)(-\sin x)$

$y' = -6\ln 3\,(3^{\cos x}\sin x)$

$y' = (-\ln 729)3^{\cos x}\sin x$ (B)

(21) $W = 0.000137L^{3.18}$

$dW = (.000137)(3.18)L^{2.18}\,dL$

$= (.000137)(3.18)(32)^{2.18}(2)$

$= 1.7$

$\therefore$ error is within ± 1.7 g W

(C)

(22) $\lim\limits_{x\to 0} \dfrac{s(x)-s(0)}{x} = \lim\limits_{x\to 0} \dfrac{s(x)-s(0)}{x-0} = 1$

is $s'(0) = 1$ (D)

(23) $y = 6x^{1/3} + 3x^{4/3}$

$y' = \frac{1}{3}\cdot 6x^{-2/3} + 3\cdot\frac{4}{3}x^{1/3}$

$y' = 2x^{-2/3} + 4x^{1/3}$

$y'' = -\frac{4}{3}x^{-5/3} + \frac{4}{3}x^{-2/3}$

$-\frac{4}{3}x^{-5/3} + \frac{4}{3}x^{-2/3} = 0$

$x^5 - x^2 = 0$

$x^2(x^3-1) = 0 \Rightarrow x=0, x=1$

y'': + + + + + + (0) - - - - - (1) + + + + +

$\therefore$ (0,0) and (1,9) (B)

(24) $y = 6x^4 + 24x^3 - 540x^2 + 7$

$y' = 24x^3 + 72x^2 - 1080x$

$y'' = 72x^2 + 144x - 1080 = 0$

$72(x+5)(x-3) = 0 \Rightarrow \{3,-5\}$ (D)

(25) $f(x) = 2x^3 - 6x$

$f'(x) = 6x^2 - 6 = 0$

$x = \pm 1$ with $c = 1$

The only value on $[0,\sqrt{3}]$

(C)

(26) $\int(+3\sin x + 5\cos x)\,dx$

$= -3\cos x + 5\sin x + C$

(C)

(27) $y = x^{5/3} + x^{1/3}$

Any line $\perp$ to $x + 2y = 7$ has slope $= 2$.

$y' = \frac{5}{3}x^{2/3} + \frac{1}{3}x^{-2/3}$

$\frac{5}{3}x^{2/3} + \frac{1}{3}x^{-2/3} = 2$

$5x^{4/3} - 6x^{2/3} + 1 = 0$

$x = 1,\ x = \left(\frac{1}{5}\right)^{3/2}$

$(1,2)$ and $\left(\frac{\sqrt{5}}{25}, \frac{26\sqrt{5}}{125}\right)$ (B)

(28) $f(x) = g(h(x))$

$f'(x) = g'(h(x))\cdot h'(x)$

$f'(5) = g'(h(5))\cdot h'(5)$

$f'(5) = g'(3)\cdot(-2)$

$f'(5) = (4)(-2) = -8$

(A)

(29) $V = lwh = 900$

$V = \frac{3}{4}l\cdot l\cdot h = \frac{3}{4}l^2h \Rightarrow h = \frac{1200}{l^2}$

$C = 3l^2 + \frac{9}{4}l^2 + 2\cdot 6\left(\frac{3}{4}l\right)\left(\frac{1200}{l^2}\right) + 2\cdot 6(l)\left(\frac{1200}{l^2}\right)$

$C = 3l^2 + \frac{9}{4}l^2 + 25200l^{-1}$

$C' = 6l + \frac{9}{2}l - 25200l^{-2} = 0$

$l = \sqrt[3]{2400}$

$\therefore$ minimized cost yields dimensions (A)

$(l,w,h) = \left(\sqrt[3]{2400}, \frac{3\sqrt[3]{2400}}{4}, \frac{\sqrt[3]{2400}}{2}\right)$

(30) (D)