Individual

1. A
2. C
3. D
4. A
5. B
6. D
7. A
8. C
9. C
10. B
11. B
12. E
13. B
14. C
15. D
16. A
17. C
18. C
19. C
20. B
21. B
22. B
23. E
24. A
25. E
26. C
27. C
28. D
29. A
30. B

Team

1. -3
2. 43
3. 1
4. 4
5. 2
6. 2
7. 1000
8. 12
9. -4
10. 8
11. 3
12. 1
13. 2
14. 7
15. 240

22. Answer: B. $\frac{1}{3}$
Let A be the event that the elder child is female and let B be the event that at least one child is male. The set of all possible equally likely states can be written as {BB, BG, GB, GG}. Then A ={GB, GG} and B ={BB, BG, GB}. Finally, $A \cap B$ ={GB}. We are asked for the probability of A given B, which is found by $P(A \backslash B) = P(A \cap B)/P(B) = \frac{1}{4}/\frac{3}{4} = \frac{1}{3}$. In other words, we may think that there is 1 way the family can have an elder female and at least one male, while there are 3 ways the family can have at least one male, so the answer is a third.

23. Answer: E. 4
Inspection, Cramer's rule, or any other method will reveal that $x = 0$, $y = 4$ is the solution, so that $x + y = 4$.

24. Answer: A. half-line
We rewrite the equation as $\log_2 y = \log_2 4 + \log_2 x = \log_2(4x)$ so that $y = 4x$. But the logs are only defined when $x, y > 0$ so that the line is only defined in the first quadrant, making it a half-line because it only extends to infinity in one direction. Lines, on the other hand, must extend to infinity in both directions and rays must have an end point.

25. Answer: E. 2
By long division or synthetic division we find that

$$\frac{x^2+3x+4}{x+2} = x+1+\frac{2}{x+2}.$$

This can be verified by multiplying back, $(x+1+\frac{2}{x+2})(x+2) = (x+1)(x+2)+2 = x^2+3x+4$.

26. Answer: C. 1000
$|z^6| = |z|^6$. We find that $|z| = \sqrt{10}$ so that $|z^6| = 1000$.

27. Answer: C. 2
We simply substitute the function definition into $f(1-x)+f(x) = \frac{2(1-x)}{2(1-x)-1} + \frac{2x}{2x-1} = \frac{2-2x}{1-2x} + \frac{2x}{2x-1} = \frac{2-4x}{1-2x} = 2$.

28. Answer: D. 16
The solution so the inequality is $4x + 2 > x + 8$ or $-4x - 2 > x + 8$ - a disjunction. Thus, we solve each individually to find $x > 2$ or $x < -2$. Since these are strict inequalities, that leaves -10,-9,-8,-7,-6,-5,-4,-3,3,4,5,6,7,8,9,10, which is 16 (all but 5 of the 21).

29. Answer: A. -3
The characteristic of $y = \log x$ is defined as $\lfloor y \rfloor$, the greatest integer not exceeding the value of the log. The mantissa is always a nonnegative real number less than one, so that the characteristic added to the mantissa is equal to the value of the log. $\log_{10}(\frac{1}{128} \approx -2.107$ so that the characteristic is -3.

30. Answer: B. 12
We must find the number of factors of 10 in the factorial. Since there are far more factors of 2 than factors of 5, we must count the factors of 5. Only 5, 10, 15, ..., 50 have factors of 5. There are 10 such numbers. Moreover, 25 and 50 have an extra factor of 5, which brings us up to 12. Thus, there are twelve factors of 10 in 50! and therefore 50! ends in 12 zeros.

Team

1. Answer: -3
For $(h, k), r$, we complete the squares to get $(x+2)^2 + (y+3)^2 = 16$ so that $h = -2$, $k = -3$, and $r = 4$.
For d, we rewrite the parabola in the form $-4(x + \frac{5}{4}) = y^2$ so that $4d = 4$ and thus $d = 1$.
Finally $m = -\frac{A}{B} = -\frac{6}{2} = -3$.
Thus, $h + k + r + d + m = -3$.

2. Answer: 43
We could find the roots to be 2, -2, $\frac{3}{2}$ and apply the requested operations to them, or we could use algebra. Let a, b, c be the roots. We know that $a + b + c = -\frac{3}{2}$, $ab + bc + ac = -4$, and $abc = -6$. Thus, $R = \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{bc+ac+ab}{abc} = \frac{2}{3}$, and $S = a^2+b^2+c^2 = (a+b+c)^2 - 2(ab+bc+ac) = \frac{9}{4} + 8 = \frac{41}{4}$. Therefore, $3R + 4S = 43$.

3. Answer: 1
Letting $x = 1$ in the definition for g we get $g(1 \cdot y) = g(1) + g(y)$ so that $g(1) = 0$. Now we let $x = 0$ in the definition for f to get $f(0 + y) = f(0)f(y)$. Since $f(y) \neq 0$, we conclude that $f(0) = 1$. Thus, $f(g(1)) = 1$.

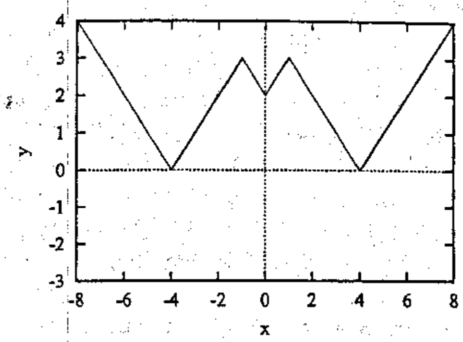
4. Answer: 4
For A, $xy = 1$ lies only in the first and third quadrants while $x^2 - 2x + y^2 + 2y + 1 = 0$, which is equivalent to $(x-1)^2 + (y+1)^2 = 1$, lies only in the fourth quadrant. Since $xy = 1$ never touches the axes, there can be no intersection points so $A = 0$.
For B, we find that $x^2 + 2x + y^2 = 0$ is a radius 1 circle at $(-1, 0)$ and $x^2 - y^2 = 1$ is a hyperbola with vertices at $(-1, 0)$ and $(1, 0)$. Therefore, we conclude that there must be exactly two intersection points so $B = 2$.
Thus, $A + B^2 = 4$.

5. Answer: 2
I. T - $f(x) = x^2 + 3$ is an even function.
II. F - $f(x) = \log x$ is not defined for negative x so it can't be even.
III. F - $x^2 + 2y^2 = 1$ is an ellipse.
IV. F - $2x^2 - 3y + 4 = 0$ has a latus rectum $\frac{3}{2}$ units long.
V. F - $f(x) = |x|$ is not one-to-one.
VI. T - $1 - x^9 = (1 - x)(1 + x + x^2 + \ldots + x^8)$, so $(1 - x)$ is indeed a divisor.
Thus there are 2 true statements.

6. Answer: 2
Graph the function from inside out. The graph of

$$\left|\left||x| - 1\right| - 3\right|$$

is shown below. Thus, subtracting $B = 2$ is the only way to get five roots.



7. Answer: 1000
For arithmetic sequences, $a_n = a_1 + (n-1)d$, so that $A = 1 + 33(3) = 100$.
For arithmetic series, $\sum_{j=1}^{n} a_j = \frac{n}{2}(a_1 + a_n)$, so that $B = \frac{21}{2}(2 + 82) = (21)(42) = 882$.
For geometric sequences, $a_n = a_1 r^{n-1}$, so that $C = \left(\sqrt{2}\right)^8 = 16$.
For geometric series, $\sum_{j=1}^{\infty} = \frac{a_1}{1-r}$, so that $D = \frac{2/3}{1/3} = 2$.
Thus, $A + B + C + D = 1000$

8. Answer: 12
$\sigma(196) = 1 + 2 + 4 + 7 + 14 + 28 + 49 + 98 + 196 = (1 + 2 + 4)(1 + 7 + 49) = 399$, $\sigma(4) = 1 + 2 + 4 = 7$, and $\sigma(49) = 1 + 7 + 49 = 57$, so that $\frac{\sigma(196)}{\sigma(4)\sigma(49)} = 1$.
$\sigma(9) = 1 + 3 + 9 = 13$, $\sigma(25) = 1 + 5 + 25 = 31$, and $\sigma(225) = 1 + 3 + 9 + 5 + 15 + 45 + 25 + 75 + 225 = (1 + 3 + 9)(1 + 5 + 25) = 403$, so that $\frac{\sigma(9)\sigma(25)}{\sigma(225)} = 1$.
$\sigma(11) = 1 + 11 = 12$ so that the final answer is 12.
Note that sigma is a multiplicative function which is why the first two quotients are one.

9. Answer: -4
A is the sum of the roots of $x^2 + 5x - 36$, so $A = -5$.
B is the solution of $\log_2(x^2) = A + 1$ so that $B = 2^{(A+1)/2} = \frac{1}{4}$.
Finally, C is the slope of $x + By = 2$ so $C = -\frac{1}{B} = -4$.

10. Answer: 8
We can solve explicity for $(x, y, z) = (\frac{13}{5}, \frac{4}{5}, -\frac{7}{5})$ and then evaluate $x^2 + xy - yz - z^2 = 8$, or we can multiply the first two equations together to get the exact same thing with much less work.

11. Answer: 3
For A, the requested line is $4x + 3y = 9$, which has a y-intercept of 3, so $A = 3$.
For B, the requested line is $2x + 3y = -4$, which has an x-intercept of -2, so $B = -2$.
For C, the vertex is midway between the directrix and focus so it is at $(4, 1)$. The focal distance is $a = 2$ so the equation is $8(y - 1) = (x - 4)^2$. The y-intercept of this is $y = 3$.
Thus, $(A + B)(C) = 3$.

12. Answer: 1
We set $x = 1 + \log_2(-1 + 2^x)$ and attempt to solve. Thus, $2^{x-1} = 2^x - 1$. We can multiply by 2^{1-x} to get $1 = 2 - 2^{1-x}$. Thus, $2^{1-x} = 1$, so that $1 - x = 0$ and thus $x = 1$. This answer is consistant with the problem statement.

13. Answer: 2
We need

$$\det \begin{pmatrix} 1-\lambda & 3 \\ 3 & 1-\lambda \end{pmatrix} = 0.$$

This gives the quadratic equation $(1-\lambda)^2 = 9$ so that $1 - \lambda = \pm 3$ and thus $\lambda = 1 \pm 3$. Therefore the sum of both possible values of λ is 2.

14. Answer: 7
Set A contains only -4 and 2. Set B is the set $\{x : x \leq -3$
Or $x \geq 3$. the integers are $-5,-4,-3, 2, 3, 4,$ and 5

15. Answer: 240
$W = 5!4!$.
$X = \frac{6!}{2}$. This is 6! instead of 5! because the clasp makes the choice of the first key important, and it is divided by two because the key ring can be flipped.
$Y = \frac{12!}{3!2!2!}$.
$Z = \frac{12!}{6!}$.
Thus, $\frac{WY}{XZ} = \frac{5!4!12!2!6!}{6!12!3!2!2!} = 2(5!) = 240$.