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Algebra II Team Questions
January Regional – Leto '99

1. Given that $x = -2^4$ and $y = 9^{\frac{2}{3}}$ and $z = 3^{\frac{1}{3}}$ the value of the product xyz, when simplified, equals $a\sqrt[3]{b}$. Find the value of a + b.

2. The x-intercept of the line given by $f(x) = 2x - 1$ is k. For the line given by f(x-1), the x-intercept is ck. Give the value of c.

3. If $\begin{vmatrix} 2 & 6 & 8 \\ 3 & 4 & 5 \\ -1 & 9 & (k+1) \end{vmatrix} = 5$ give the value of k.

4. Given that $\frac{1}{x} + \frac{2}{x+2} = \frac{5}{x+3}$ then give the value of $2x^2 - x + 12$.

5. Of the roots of the polynomial function $y = 4x^4 - 61x^3 + 158x^2 + 361x - 462$, one such root is 11. Let k be the sum of the other roots. Give the value of $\frac{1}{k+1}$ in fraction form.

6. For c, a positive Real Number $42^x = c^2$ and $6^x = 4$ and $7^x = 5$. Give the value of $\frac{1}{c-1}$.

7. If it is known that $\log_2(\log_3 m) = 4$ then let A be the number of positive integral factors of m. If $y = ((((x-1)^2)^3)^4)^9$ and $\log(x) = 0$ then let B be the value of $(2x)^y$. Find the value of $A - B$.

8. I have won 200 games of a computer game and lost 54 games. The computer says that I have won 79% of the games played. It rounds to the nearest whole number. Find the minimum number extra games must I play (in addition to the original 200), with no losses, to have the computer say I have won 80%? Let this number be P.
If we again refer to the original 200 games played, find the minimum extra games I must play, assuming I will lose $\frac{1}{10}$ of these extra games, to have the computer say I have won 80%. Let this number be Q. Give the value of P+Q.

9. For f(x)=x+3, $\frac{A}{f(x)} + \frac{B}{f(x+1)} = \frac{1}{f(x)\cdot f(x+1)}$ for $x \neq -4, -3$. Give the value of A-B.

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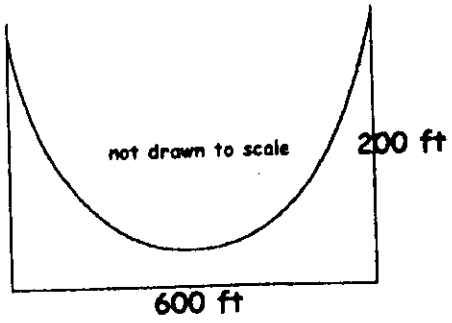
10. A cable hangs in the form of a parabolic curve from two supports, each 200 feet high. The supports are 600 feet apart, and the cable hangs 40 feet above ground at its lowest point. At a point 10 feet (horizontal distance) from a support, how high above ground is the cable? Disregard the thickness of the cable in your calculations and round to the nearest hundredth of a foot.



11. The function f is an even function such that $f(2) = 3$.
The function g is an odd function such that $g(5) = 8$.
If $a\#b$ is defined as $a^2 - 2ab$ then give the value of $(f(-2)) \# (g(-5))$.

12. If $\log 2 = x$ and $\log 3 = y$ then let
A = the value of $\log 30$ in terms of x and/or y.
B = the value of k which makes the equation $2x + 3y = \log(9k)$ true.
C = the value of $\dfrac{\log_2 3}{\log_2 10}$ in terms of x and/or y. Give the value of (A + B - C).

13. A curve is such that all points on it are equidistant from the line x=5 and the point (7, -1). What is the abscissa of the point on the curve whose ordinate is 0 ?

14. Two room heaters are considered for purchase. Heater A costs $120 and will cost $5 per day to operate. Heater B costs $180 and will cost $3 per day to operate. Assuming that the heaters will be used every day, let D be the minimum number of whole days that heater B must be used, considering purchase price and operation costs, to have an overall cost of less than heater A. On day D, how many dollars will heater B have cost?

15. $f(x) = 2x + 6$ and $g(x) = x^2$.
$f^{-1}(x)$ denotes the inverse function to f.
Find the value of $f^{-1}(2) - (f(2))^{-1} + g(f(2))$.