

ALGEBRA 2

TEAM QUESTION #1

JANUARY, 1998

Each die of a given pair is defective. One of them has two threes and no four. The other has two fours and no three.

If this pair is tossed, what is the probability that the sum that shows is odd? Give your answer as a reduced fraction.

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TEAM QUESTION #2

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Solve for x:
$$\frac{x^2 + 5x + 6}{x^2 - 2x - 8} = \frac{x^2 - 4x - 5}{x^2 - 8x + 15}$$

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TEAM QUESTION #3

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Find the quadratic function whose graph has a vertex at (3,4) and contains the point (1,2).

Write your answer as $f(x) =$

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TEAM QUESTION #4

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Consider the graph of $x^2 + y^2 = 25$.

How many points on this graph have points (a,b) where both a and b are integers?

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TEAM QUESTION #5

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Express the number: $\overline{19.9819}$ as an improper reduced fraction.

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TEAM QUESTION #6

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Find all real number solutions for the equation:

$$\sqrt{x} + 6x\sqrt{x} = 5x$$

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TEAM QUESTION #7

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Given: A = the number of final zeros when 1998! is expanded.

B = the number of positive integral divisors of 1998

Find A + B

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TEAM QUESTION #8

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Given: $f(x) = \begin{cases} x^2 & \text{for } x < 3 \\ x & \text{for } x \geq 3 \end{cases}$

$$g(x) = \begin{cases} -x & \text{for } 0 \leq x \leq 4 \\ \sqrt{x-3} & \text{for } x > 4 \end{cases}$$

$$h(x) = \begin{cases} 4 - x + x^2 & \text{for } x < 2 \\ 1 - 2x + 3x^2 & \text{for } x \geq 2 \end{cases}$$

$$A = f(g(h(1))) \quad \text{and} \quad B = f(g(h(2)))$$

What is the value of $\sqrt{A+B+3}$? [Round your answer to the nearest integer.]

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TEAM QUESTION #9

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There exists a complex number, $a + bi$, such that the number equals the square of its conjugate. In fact, there are two such numbers!

What are these two numbers?

Showme High School has a bunch of math students with special skills. 56 can square a circle, 30 can trisect an angle, 36 can double a cube, 10 can square a circle and trisect an angle, 13 can square a circle and double a cube, 11 can trisect an angle and double a cube, and 8 can do all three tasks. Another 22 students can divide by 0, but do not have the other three special skills. The remaining 602 students cannot do any of those things.

How many math students are enrolled in Showme High School?

[Note: The teachers can't do any of that stuff either!]

Expand and simplify:

$$(a + b + c)^2 + (a + b - c)^2 + (a - b + c)^2 + (-a + b + c)^2$$

Find the ordered triple (x, y, z) that is the solution of the system:

$$\frac{5}{x} + \frac{2}{y} + \frac{2}{z} = 2$$

$$\frac{1}{x} + \frac{2}{y} - \frac{2}{z} = -1$$

$$\frac{3}{x} + \frac{1}{y} - \frac{4}{z} = 0$$

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TEAM QUESTION #13

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Solve for x:
$$\frac{3^x - 3^{-x}}{3^x + 3^{-x}} = \frac{1}{2}$$

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TEAM QUESTION #14

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Suppose 50 items can be sold for \$60 apiece. If the price increases by \$10, then the number of articles that sell decreases by 5. What selling price will yield the maximum gross sales?

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TEAM QUESTION #15

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A chemist has 7 liters of a solution which is 35% antifreeze and 65% water. He needs 7 liters of solution which is 50% antifreeze and 50% water. How many liters should he drain from the container and replace with antifreeze? Give your answer correct to the nearest hundredth.