

JANUARY 23, 1993

ALGEBRA II INDIVIDUAL TEST

REGIONAL COMPETITION

The abbreviation "NOTA" represents "None of these answers."

1. Let  $f(x) = \frac{1}{3}$  of  $g(x)$   
 $g(x) = \frac{1}{2}$  of  $h(x)$  and  
 $h(x) = \frac{1}{4}$  of  $x$ .

Evaluate  $f(120)$ .

- A) 30    B) 10    C) 180    D) 5    E) NOTA

2. If  $a*b$  represents  $\frac{2b-a}{a^3-8b^3}$  then  $a*a=?$

- A)  $7a^2$     B)  $-6a^2$     C)  $\frac{-1}{7a^2}$     D)  $\frac{-1}{6a^3}$     E) NOTA

3. Give the binomial factor of  $27x^6-1$ , when factored over the set of integers.

- A)  $3x^3-1$     B)  $3x^3-1$     C)  $3x^2+1$     D)  $3x^2-1$     E) NOTA

4. For  $f(x)=3^x$ , then  $f(x+2)-f(x) = ?$

- A) 3    B)  $3f(x)$     C)  $4f(x)$     D)  $8f(x)$     E) NOTA

5. A pharmacist wishes to strengthen a mixture that is 10% alcohol to one that is 30% alcohol. How much pure alcohol should be added to 7 liters of the 10% mixture?

- A) 2 liters    B) 1.4 liters    C) 7 liters    D) 1.6 liters    E) NOTA

6. Simplify:  $\frac{(\sqrt[3]{3} - \sqrt[3]{5})(\sqrt[3]{9} + \sqrt[3]{15} + \sqrt[3]{25})}{(\sqrt[3]{3} + \sqrt[3]{5})(\sqrt[3]{9} - \sqrt[3]{15} + \sqrt[3]{25})}$

- A)  $\frac{1}{2}$     B) 1    C)  $-\frac{1}{4}$     D) -15    E) NOTA

7. If  $2 - \sqrt{x-1} = \sqrt{x}$ , what is the value of  $(\sqrt{x} - 4x + 1)$ ?

- A) 25    B) 0.2    C) 4    D) -4    E) NOTA

8. Simplify:  $\frac{xy^{-1} + yx^{-1}}{x^2 + y^2}$

- A)  $xy$       B)  $\frac{1}{xy}$       C)  $\frac{1}{x^2 + y^2}$       D)  $x^2 + y^2$       E) NOTA

9.  $\begin{bmatrix} 2 & -5 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 20 \\ 7 \end{bmatrix}$

Find the sum,  $x+y$ .

- A) 3      B) -2      C)  $\frac{20}{3}$       D) -1      E) NOTA

10. The sum of the roots of  $3x^3 - 4x^2 + x - 1$  is ...

- A)  $\frac{4}{3}$       B)  $-\frac{4}{3}$       C)  $\frac{1}{3}$       D)  $-\frac{1}{3}$       E) NOTA

11. Given that  $f(x) = \log_4(5x+10)$  and  $g(x) = \log_5(21x+1)$ , find  $(fg)(3)$ .

- A) 80      B) 6      C) 5      D) 20      E) NOTA

12.  $\sqrt{x\sqrt{x}} =$

- A)  $x^{\frac{1}{2}}$       B)  $x^{\frac{1}{4}}$       C)  $x^{\frac{3}{4}}$       D)  $x$       E) NOTA

13. If the solutions of  $f(x)=0$  are -2 and 4, then which of the following is false?

- A) The solutions of  $f(x-1)=0$  are -1 and 5.  
 B) The solutions of  $f(x+1)=0$  are -3 and 3.  
 C) The solutions of  $f(-x)=0$  are -4 and 2.  
 D) The solutions of  $f(2x)=0$  are -4 and 8.  
 E) All are true (NOTA).

14. Which is NOT a factor of  $3^{12} - 2^{12}$ ?

- A) 5      B) 7      C) 19      D) 61      E) all are factors of  $3^{12} - 2^{12}$

15. Give the range of  $y = |3x+12| + 2$ .

- A)  $y \geq 0$       B)  $y \geq 2$       C)  $y \geq -4$       D)  $y \geq 4$       E) NOTA



23. Given  $f(x)=2x+1$ , what is  $\frac{1}{f^{-1}(2)}$  if  $f^{-1}(x)$  represents the inverse of  $f$ ?
- A) 2      B)  $\frac{1}{2}$       C)  $\frac{1}{5}$       D)  $-\frac{1}{5}$       E) NOTA
24. Student X can solve a work problem in 5 minutes and Student Y can solve the same problem in 4 minutes. Exactly how many minutes will it take them working together if they stay at their usual individual rates?
- A) 4.5 minutes      B) 3.5 minutes      C) 2 minutes  
D) less than 1 minute      E) NOTA
25. Which statement is NOT necessarily true about the function  $f(x) = ax^3 - bx$  ( $a \neq 0, b \neq 0$ )?
- A) The graph of  $f$  passes through the origin.  
B) The equation  $f(x)=0$  has three solutions.  
C) The graph of  $f$  passes through the x-axis three times.  
D) The graph of  $f$  passes through the y-axis once only.  
E) All are true (NOTA).
26.  $|3-5i| = ?$  ( $i = \sqrt{-1}$ )
- A) 4      B)  $3+5i$       C)  $5i-3$       D)  $\sqrt{34}$       E) NOTA
27. If  $i = \sqrt{-1}$ , then  $(i)^{2001} = ?$
- A) 1      B) -1      C) i      D) -i      E) NOTA
28. What is the product of the solutions of the equation  $2^{2x+1} - 17 \cdot 2^x + 2 = 0$ ?
- A) 2      B)  $-\frac{1}{2}$       C) -3      D)  $-\frac{1}{4}$       E) NOTA
29. What is the  $A_{21}$  term of  $A = \begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 4 & 0 \\ 1 & -1 \end{bmatrix}$ ?
- A) 10      B) 6      C) 2      D) 8      E) NOTA
30. The difference between the squares of two consecutive positive integers is 7. The greater number is ...
- A) 3      B) 4      C) 6      D) 7      E) NOTA