Algebra 1 Team Regional January

1) Three different pizza companies competed to get the school's cafeteria business. Each had a different price plan and your job is to pick the one that would be the best value for the school and tell the price. The best value is defined as getting the most pizza per square inch. Company A has a pizza that is 14 inches in diameter with a price of $ 12. Company B is offering a pizza with a diameter of 15 inches for $ 14. Company C is offering two 9 inch diameter pizzas for $ 10. Which company is offering the best value and what is the value in dollars per square inch? Round your answer to the nearest thousandth place.

2) Find the sum of the mean, the median, and the mode for the following set of numbers.
{ 12, 15, 18, 12, 19, 22, 18, 12, 28, 32, 42, 32, 12, 34, 34, 82, 66, 2001 }

3) Simplify the following expression in the lowest terms possible:

$$-2R\{-4(3-6\{-7+11(6-13\{5-3\}-4\{-6-4\})-6\})-3\}$$

4) Find X + Y for the following system of equations with the given information:

X = 4B - 2A
B = 2A - 4C
C = -32 + 2D
D = 32 + C

Y = 5(6 - 9)(3 - F) - 3B
F = 16(4 + G)
G = - | 9 - 13 | + H
H = 2001(2002) - (2002)2001

5) In football, a touchdown is worth six points, a fieldgoal is worth three points, a safety is worth 2 points, and an extra point is worth only one point. If an excellent team, such as the Dolphins, were to score 36 points with exactly 2 of them being scored as extra points, then list all of the different point combinations. The number of safety's are to be equal to or less than the number of touchdown's. (Note: The answer(s) needs to be given as an ordered quadruple(s): (touchdown(s), fieldgoal(s), safety(s), extra point(s)).

6) Imagine if you will, a right triangle. Isn't it a beautiful thing, 3 perfectly straight line segments, one absolutely awesome ninety degree angle and two cute little acute angles. We know that this triangle is not an isosceles triangle. The absolute value for the difference between the smallest side and the hypotenuse is a perfect square. The sum of all three sides are less than eight squared. The sum of the cubes of the two legs is greater than 3000 and less than 10,000. If the triangle is a Pythagorean Triple then find the area of this right triangle.

7) x, y, and z are all natural numbers. $x > y > z$, x is between 12 and 45 and is a prime number, y is the sum of the positive integral factors of twenty, z is the absolute value of the number that is added to (2001 - 1991) to obtain zero. Find the sum of the squares of x, y, and z.

8) Assume no variables are equal to zero. Simplify:

$$\frac{15x^2z^4}{16y^3z^2} \bullet \frac{x^3+28x^2-(7x)(4x)}{5x^5-5x^3} \div \frac{3x^2z^2-3xyz^2+3y^2z^2}{8x^2-8}$$

Geometry Individual Test

January Regional competition 2005

Solutions to individual test

D 1. $2x+3x+4x=180, x=20$. angles are 40, 60, 80. Smallest exterior angle is the supplement of the largest interior angle which would be 100.

C 2. Ratio of sides is 14:28 or 1:2. Ratio of perimeters is 1:2. Perimeter of ABC=29, $\frac{1}{2}=\frac{29}{x}$, x=58.

B 3. Since lines are parallel, $\angle AGH \cong \angle GHD$ Because they're alternate interior angles. Therefore, $3x-10=80, x=30, 2x=60$.

C 4. $\triangle ADE \sim \triangle ABC$ by AA. Ratio of sides is 1:2. Let AE=x, $\frac{x}{6}=\frac{1}{2}, x=3$ = AE, EC=3. $\frac{DE}{8}=\frac{1}{2}$, DE=4. Since the triangle is right use Pythag to find AB=10, AD=BD=5. Perimeter of trapezoid = 3+8+5+4=20.

D 5. $\angle A$ and $\angle B$ are supplementary because they are consecutive angles. $3x-1+2x+11=180$, x=34. $\angle C \cong \angle A, m\angle A=101$

A 6. $m\angle B=x, m\angle A=2x, m\angle C=3x+6$, $6x+6=180$, $x=29$

C 7. $AD:ED=2:1$, since AD=18, ED=6. Use Pythag to find EC= 10

C 8. $40+x+20+30=180, x=90$

C 9. The width is 15. It's a Pythag triple, so Area is 15x8=120.

C 10. Area of original is 100π. Radius Decreased by 2 is 8 making area 64π. Decrease in area is 36π. $36\pi:100\pi=36\%$ decrease.

D 11. $\angle FDP \cong \angle EBP$ - alternate interiors $\angle DPF \cong \angle BPE$ - vertical angles $\triangle DPF \sim \triangle BPE$ by AA. Let DC=4x, which makes DF=2x. Since $\overline{DC} \cong \overline{AB}$, AB=4x and since AE=1/4 AB, AE=x and EB=3x. DF:BE=DP:BP=2:3.

A 12. $180-x=2(180-(90-x))-78, x=26$

A 13. Using Pythag on $\triangle ABC$, AC-21. Using Pythag on $\triangle ABD$, AD=15. AC-AD=6

D 14. Let AM = x, FM =FA= $3x$. Since Perimeter is 42, $7x=42$, x=6. FA=18

A 15. 64>36 + 9, triangle is obtuse.

B 16. Use vertical angles for $\angle 1$, then the 3 Angles are supplementary which makes $m\angle 1=180-a-b$

A 17. I always, II never, III sometimes

D 18. $m\angle MPN=m\angle MNP=50$, $m\angle PMN=80, m\angle TMN=50$ by alternate interior angles. $m\angle TMP=m\angle PMN+m\angle TMN=80+50=130$.

A 19. Draw the altitude from point A and let this be point E.. This is also 6. Since $m\angle D=30$, AD=12 and DE=$6\sqrt{3}$. $\sqrt{192}=8\sqrt{3}$. So DC=$14\sqrt{3}$.

B 20. Let BD=x, AB=x+4. In $\triangle CBD$, BD=x, BC=$x\sqrt{3}$, CD=2x. Using $\triangle ABC$, Since BC=$x\sqrt{3}$, AB=3x. We have two expressions for AB: $3x=x+4, x=2$, AB=6,CB=$2\sqrt{3}$.

A 21. $m\angle CEF=30, m\angle BEF=50$, 50-30=20.