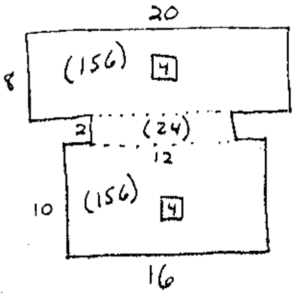
Geometry Team Solutions

Feb. 8, 1992

1)



$$\begin{array}{r} 156 \\ 156 \\ 24 \\ \hline 336 \end{array}$$

Area is 336

2) $\frac{(n-2)\cdot 180}{n}$

$\frac{(102-2)\cdot 180}{102}$

$\frac{100\cdot 180}{102}$

$\frac{18{,}000}{102} = \frac{3000}{17}$

3)

square

diagonal $=10$

side $= 5\sqrt{2}$

Area $= (5\sqrt{2})^2 = 50$

4) $n = \frac{9\cdot 6}{2} = 27$

$s = 9$

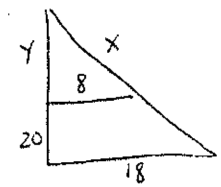
pattern for r:

# of lines	0	1	2	3	4	5
# of regions	1	2	4	7	11	16

$r = 16$

$9 + 16 - 27 = -2$

5)



$\frac{4}{9} = \frac{y}{y+20}$

$9y = 4y + 80$

$5y = 80$

$y = 16$

by P.T.

$16^2 + 8^2 = x^2$

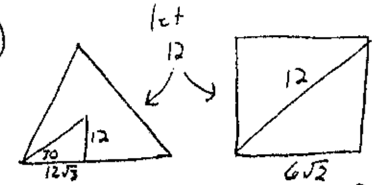
$256 + 64 = x^2$

$x^2 = 320$

$x = 8\sqrt{5}$

Geom. Team Solutions (cont.)

6)



$A = \frac{1}{2}ap$

$A = \frac{1}{2}\cdot 12 \cdot 72\sqrt{3}$

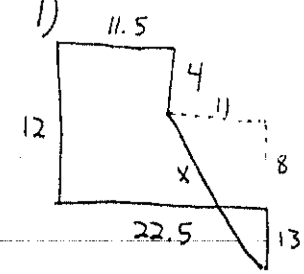
$A = 432\sqrt{3}$

$A = (6\sqrt{2})^2$

$A = 72$

$\frac{432\sqrt{3}}{72} = \boxed{6\sqrt{3} : 1}$

7)



$x^2 = 11^2 + 21^2$

$x^2 = 121 + 441$

$x^2 = 562$

$x = \sqrt{562}$

8) $hyp = \sqrt{24^2 + 32^2}$

$hyp = \sqrt{576 + 1024}$

$hyp = \sqrt{1600}$

$hyp. = 40$

Area of $\Delta = \frac{1}{2}bh$

$= \frac{1}{2}\cdot 24 \cdot 32$

$= 384$

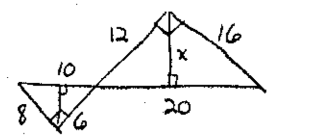
$\frac{1}{2}\cdot 40 \cdot x = 384$

$20x = 384$

$x = \frac{96}{5}$

or 19.2

9)



Area of large $\Delta = \frac{1}{2}\cdot 12 \cdot 16$

$= 96$

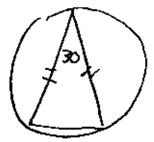
Area also $= \frac{1}{2}\cdot 20 \cdot x$

$96 = 10x$

$x = 9.6$ or $\frac{48}{5}$

Geom Team Solutions (cont.)

10)



$\ell = 25$

measure $= 60$

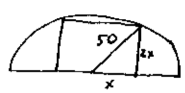
25 is "$\frac{1}{6}$" of 150

$C = 150$

$\pi d = 150$

$d = \frac{150}{\pi}$

11)



$x^2 + 4x^2 = 2500$

$5x^2 = 2500$

$x^2 = 500$

$x = 10\sqrt{5}$

side of $\square = 20\sqrt{5}$

diagonal $= 20\sqrt{10}$

12) $\overset{\ell}{(7x)^2} + \overset{w}{(5x)^2} + \overset{a}{x^2} = 10^2$

$100 = 49x^2 + 25x^2 + x^2$

$75x^2 = 100$

$x^2 = \frac{100}{75} = \frac{4}{3}$

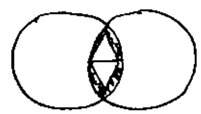
$x = \frac{2\sqrt{3}}{3}$. $w = 5x$

$w = \frac{10\sqrt{3}}{3}$

13)

since side of $\square$ is same as diam. of circle, ratio is $\frac{C}{d} = \pi$

14)



equilateral ← Δ's

area is 2 equilateral Δ's plus shaded area.

one Δ → $A = \frac{s^2\sqrt{3}}{4} = \frac{144\sqrt{3}}{4} = 36\sqrt{3}$ = area of one Δ

2 Δ's → $72\sqrt{3}$ = 2 Δ's

one ◗ $= \frac{144\pi}{6} - 36\sqrt{3}$
(sector) (triangle)

one ◗ $= 24\pi - 36\sqrt{3}$

$4 = 96\pi - 144\sqrt{3}$

Total $= 72\sqrt{3} + (96\pi - 144\sqrt{3})$

Total area of ● $= 96\pi - 72\sqrt{3}$

15) $D = \sqrt{6^2 + 15^2 + 10^2 + 9^2}$ ↖ solve to get 9

$D = \sqrt{36 + 225 + 100 + 81}$

$D = \sqrt{442}$

Pre-Calculus Team Solution Key

2-8-92

① $\frac{2\sin^2 3x}{\cos^2 3x} + 3 = \frac{3}{\cos 3x}$ $2\sin^2 3x + 3\cos^2 3x = 3\cos 3x$

$\frac{2}{3}\pi$

$2(1-\cos^2 3x) + 3\cos^2 3x - 3\cos 3x = 0$

$\cos^2 3x - 3\cos 3x + 2 = 0$

$(\cos 3x - 2)(\cos 3x - 1) = 0$

$\cos 3x = 2$ not possible; $\cos 3x = 1$ if $3x = 0, 2\pi, 4\pi \ldots$

if $0 \le x \le \pi$, then $x = 0$ or $\frac{2}{3}\pi$

sum of the solutions is $\frac{2}{3}\pi$

② $261 + 10\pi$

$A = \sqrt{9+16+144} = \underline{\underline{13}}$

B: $x + y = 16$

$x = 3y$

$(x,y) = (12, 4)$

Triangle formed by $x+y=16$, $x = 3y$, and $y = 0$ therefore has base equal to 16, and altitude equal to 4 so area is $\underline{\underline{32}}$

$B = 32$

C: total area of cube + lateral area of the cylinders — area of circular ends of the cylinders

$C = 6^3 + 2\pi(1)(6) - 2(\pi(1^2)) = 216 + 12\pi - 2\pi$

$C = 216 + 10\pi$

$A + B + C = 261 + 10\pi$

③ 162

$x^3 - 63x + k = 0$ roots are r_1, r_2, r_3 such that $r_2 = 2r_1$

$ax^3 + bx^2 + cx + d = 0$

sum of roots $= -\frac{b}{a} = 0$,

so $r_1 + r_2 + r_3 = r_1 + 2r_1 + r_3 = 0$. Therefore $r_3 = -3r_1$

sum of products of roots, two at a time is $\frac{c}{a} = -63 = r_1r_2 + r_1r_3 + r_2r_3$

$2r_1^2 - 3r_1^2 - 6r_1^2 = -7r_1^2 = -63$; $r_1^2 = 9$; $r_1 = \pm 3$

Product $= r_1r_2r_3 = -6r_1^3 = -\frac{d}{a} = -k$ so $k = 6r_1^3$

since $k > 0$, $k = 6(27) = 162$