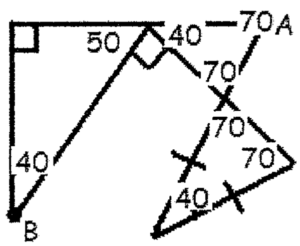
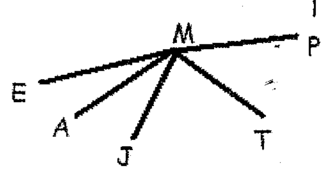
February Regional - Geometry Individual
SOLUTIONS 2005

1. (B) The radius is 3, and the area is 9π.
2. (B) The central angle equals the intercepted arc. Solving 2x+9=60-x, we find x=17.
3. (B) The perimeter of the rectangle is 64, and its area is 240. The perimeter of the square is also 64, and its area is 256. 240/256 = 15/16.
4. (E) The correct expression is $5b^2+5b$, which is not given.
5. (B) Solving $1.5s^2\sqrt{3}=18\sqrt{3}$, we find that $s^2=12$, and $s=2\sqrt{3}$.
6. (C) $\overline{MN}$ must be parallel to $\overline{BC}$ by definition of a trapezoid – so we have Similar triangles. Let x = AC. Solving $\frac{12}{18}=\frac{30}{x}$, find that x=45.
7. (D) The correct angle measurements are shown in the diagram to the right.



8. (D) let x = MY. Solving 12(15) = 18x, find x = 10. Add 18 to find XY.
9. (A) cos(D) =Adj/Hyp = 8/17.
10. (A) The small triangle is 3-4-5, and has a perimeter of 12. BC corresponds with EF. Solving $\frac{12}{4}=\frac{x}{12}$, the perimeter is 36.
11. (C) Solving x = 15 + ½ (180 – x), yields x=70.
12. (B) Only II is true.
13. (B) The area is half the product of the diagonals. So the product of the diagonals $=2(28\sqrt{7})=56\sqrt{7}$
14. (E) All yield integral second legs.
15. (B) The sum of the angles is 540. Solving, x = 35. $m\angle EST=x+40=75$.
16. (C) Solving the system (3x + 2y = 40 ; 2x + y = 20) yields x = 0, and y = 20. x + y = 20.
17. (B) The sequence is generated according to the formula $a(n+1)=a(n)+n^2$. The next term is $57+(6)^2=93$.
18. (A) $\angle AMT$ is the sum of $\angle AMJ+\angle JMT=30$. $\angle JMT$ is twice as big as $\angle AMJ$, so $\angle JMT$=20. $\angle TMP=\angle JMT=20$. The sum of these two angles is the answer, 40.



19. (B) $d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}=\sqrt{4^2+2^2+7^2}=\sqrt{69}$.
20. (D) Half the chord forms a right triangle, with the distance as one leg and the radius as the hypotenuse. $x^2+64=144$. $x=\sqrt{80}=4\sqrt{5}$. the whole chord $=2x=8\sqrt{5}$.
21. (A) $\angle EAR=\angle UAD=(180-95)=85$. $\angle ERG=85+35=120$.
22. (A) The triangle has a height of 3 and a base of 9. The area is 13.5.
23. (A). At 9:58, $60(9)+58=598$ minutes have passed. The hour hand is at $\frac{(360°)}{60(12)}\bullet 598=299°$. The minute hand is at $\frac{360°}{60}\bullet 58=348°$. The angle between them is (348-299) = 49 degrees.
24. (A) The height of the trap is $\frac{6}{\sqrt{2}}=3\sqrt{2}$. So, $A=\frac{1}{2}(3\sqrt{2})(10+20)=45\sqrt{2}$.
25. (B) The angle of the hexagon is $120°$, so the arc walked is $240°$. L= $\frac{240}{360}(2\pi(50))\approx 209$.
26. (D) Let x be the short leg. Solving $\frac{x^2\sqrt{3}}{2}=18\sqrt{3}$, $x=6$. The perimeter $=x+x\sqrt{3}+2x\approx 28.4$
27. (D) $\overset{\frown}{AC}=360°-\overset{\frown}{CBA}=145°$. $m\angle CDA=\frac{m\overset{\frown}{AC}-m\overset{\frown}{XY}}{2}$. Solving: $35=\frac{145-m\overset{\frown}{XY}}{2}$, $m\overset{\frown}{XY}=75$.
28. (B) Three side lengths may form only one distinct triangle if they satisfy the triangle inequality rule. $\frac{7}{6}$, $\frac{2}{5}$, and $\frac{9}{11}$ do satisfy the triangle inequality rule.
29. (B) $V=\frac{1}{3}l^2h$. $64=\frac{1}{3}l^2(12)$...solving, $l=4$. The base of the pyramid is a 4 by 4 square. The radius of the square is $2\sqrt{2}$. The height and radius are the legs of a right triangle, and the lateral edge is the hypotenuse. Solving $(2\sqrt{2})^2+12^2=L^2$ yields $L=\sqrt{152}=2\sqrt{38}$.
30. (C) The two legs are congruent, and the angle between each of them is known to be a congruent – a right angle. Therefore, SAS is an equivalent proof.

Answer Key

February Regional
GEOMETRY

Individual Answers:

1) B
2) B
3) B
4) E
5) B

6) C
7) D
8) D
9) A
10) A

11) C
12) B
13) B
14) E
15) B

16) C
17) B
18) A
19) B
20) D

21) A
22) A
23) A
24) A
25) B

26) D
27) D
28) B
29) B
30) C

Team Answers:

1) 261
2) 113
3) 28
4) 876
5) 237

6) 108
7) $-12\sqrt{2}$
8) 9900
9) 36
10) 70

11) $5\sqrt{205}$
12) 960
13) 6400
14) 0
15) 31