Answers to February Regional Competition
Geometry Division

Individual

1	C
2	B
3	B
4	C
5	A
6	B
7	C
8	~~C~~ E
9	D
10	B
11	C
12	B
13	C
14	D
15	E
16	B
17	B
18	C
19	C
20	D
21	D
22	~~B~~ E
23	C
24	B
25	B
26	C
27	D
28	B
29	D
30	~~A~~ E

Team

1	608
2	58.5
3	360
4	B
5	308
6	$33\pi - 18\sqrt{3}$
7	29.4
8	108.5
9	$\frac{4x-3}{2}$
10	$1125\pi+900$
11	$4\sqrt{5}$
12	$196\sqrt{3}$
13	693
14	60
15	C

Individual Solutions – Geometry February Regional

1. Sum of angles equals 180, therefore: $3x + 7 + 4x - 3 + 2x + 5 = 180$ which reduces to $9x + 9 = 180$. Solve for x: $x = 19$. Plug $x = 19$ into the three angle formulas and sum the resulting two largest angles. $64+73 = \boxed{137 = C}$

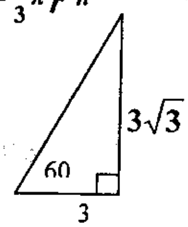
2. Call the angle "x". Therefore: $9(x - 90) = 3(x - 180) + 2$. Solve for "x". $x = 45.3333 \approx \boxed{45 = B}$

3. (AE)(CE) = (DE)(BE) when the chords intersect at a point other than the center of the circle. Plug in given lengths: (10)(CE) = (5)(6). Therefore CE = $\boxed{3 = B}$

4. Solve for radius of the circle. $\pi r^2 = 36\pi$. Therefore $r = 6$ and diameter = 12. Area of a hexagon formula is $A = 6\left(\frac{s^2\sqrt{3}}{4}\right)$.

   Plus s = 12 into formula. $\boxed{216\sqrt{3} = C}$

5. Solve for a side of the triangle. $A = \left(\frac{s^2\sqrt{3}}{4}\right)$ therefore $9\sqrt{3} = \left(\frac{s^2\sqrt{3}}{4}\right)$. S = 6. When the figure is rotated the resulting figure is a cone with radius 3 and height $3\sqrt{3}$. Use the formula for volume of a cone: $V = \frac{1}{3}\pi r^2 h$. Substitute in the values to get $V = \frac{1}{3}\pi 3^2(3\sqrt{3}) = \boxed{9\pi\sqrt{3} = A}$

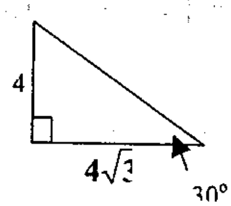


6. The ratio of the lengths of similar triangles can be squared to obtain the ratio of the areas. Therefore: $\boxed{\left(\frac{3}{12}\right)^2 = \frac{1}{16} = B}$

7. Compare the volumes of the pans to decide which is larger. Using the formula $V = d \bullet \pi r^2$ the volumes of the three pans are $A = 72\pi$, $B = 64\pi$, $C = 73.5\pi$. Therefore C is greatest. $\boxed{\text{Pan C} = C}$

8. The average of his test scores is $(379 + x) / 6$. Set the "average formula" equal to the 75 and 78 averages obtained and solve for "x", the last grade. The two "x's" are 89 and 71. The absolute value of the difference is $89 - 71 = \cancel{\boxed{18 = C}}$ E

9. Area of shaded region = (A of triangle) – (A of circle). Use 30-60-90 properties of a triangle to find the side length of the triangle = $8\sqrt{3}$ as shown below. $A = \left(\frac{\left(8\sqrt{3}\right)^2\sqrt{3}}{4}\right) - 16\pi$ Reduces to $\boxed{48\sqrt{3} - 16\pi = D}$



10. Law of Sines: $(\sin 30^\circ) / 6 = (\sin 45^\circ) / x$. "$x$" = $\boxed{6\sqrt{2} = B}$

11. Definition of linear pair. $180 - 75 = \boxed{105^\circ = C}$

12. Subtract 6 from all coordinates to shift the triangle. Resulting vertices (0,12), (0,0), (3,0). It is now on the origin with base 3 and height 12. $(0.5)(3)(12) = \boxed{18 = B}$

13. $3x + 2 + 5x + 18 = 180$. $8x + 20 = 180$. $x = \boxed{20 = C}$

14. $5 + 7 = 12$. $7 - 5 = 2$. Therefore the third side length must be $2 < x < 12$. 12 is outside this range, therefore $\boxed{D}$.

15. $\frac{20}{10} = \frac{10}{x}, \therefore x = 5$ $\quad 10^2 + 5^2 = y^2, \therefore y = 5\sqrt{5}$ $\quad \left(5\sqrt{5}\right)^2 + z^2 = 25^2, \therefore z = 10\sqrt{5}$

$x - y + z = 5 - 5\sqrt{5} + 10\sqrt{5} = 5 + 5\sqrt{5} = \boxed{E}$

16. $\left[\frac{1}{2}(b1 + b2)h\right] - [bh] = 60 - 48 = \boxed{12 = B}$

17. $\pi r^2 = 144\pi$, r =12 of the larger circle. Therefore the diagonal of the square is 24 and the side length of the square is $12\sqrt{2}$. Then the radius of the smaller circle is ½ of the side length, $6\sqrt{2}$. The area of the shaded region is $\left(12\sqrt{2}\right)^2 - \pi\left(6\sqrt{2}\right)^2 =$ $\boxed{288\text{-}72\pi = B}$

18. Definition. $\boxed{\text{Incenter} = C}$

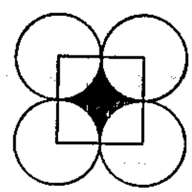
19. Logical Proof Step, $\boxed{\angle c \cong \angle d = C}$

20. Definition. $\boxed{\text{Transitive} = D}$

21. The height of the equilateral triangles enclosed in the hexagon is 12 (half of 24 since there are two). Use 30-60-90 triangle properties to find that the side length of the hexagon is $8\sqrt{3}$. Use the hexagon are formula, $A = 6\left(\frac{s^2\sqrt{3}}{4}\right)$, to find that Area $= \boxed{288\sqrt{3} = D}$

22. Area of the segment equals the area of the sector minus the area of the triangle. $\frac{\pi r^2}{4} - \frac{r^2}{2} = \frac{\pi 6^2}{4} - \frac{6^2}{2} = \boxed{9\pi - 18 = \cancel{B}}$ E

23. Radius of circles = 3. $6^2 - 4\left(\frac{\pi r^2}{4}\right) = \boxed{36 - 9\pi = C}$



24. Centroid = $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right) = \boxed{(5,3) = B}$

25. Find the difference of their position from 12. For hour hand: $180 + \left(\frac{23}{60}\right)30$. For minute hand: $\left(\frac{23}{60}\right)360$.
$191.5 - 138 = \boxed{53.5 = B}$

26. Volume of the sphere: $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi 3^3 = 36\pi$. Volume of the cone: $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi 6^2 4 = 48\pi$.
$\frac{36\pi}{48\pi} = 0.75 = \boxed{75\% = C}$

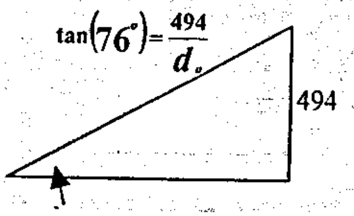
27. $\boxed{D}$

28. $x^2 + 5x = 3x + 8$. x = 2 and –4. (-4 does not produce a positive answer for 3x + 8, therefore it is an extraneous solution and should be ignored). Plug x = 2 into the two angle equations and sum. $14+14 = \boxed{28 = B}$

29. Definition of construction methods. SSS, SAS, ASA all could prove a triangle congruent, however SSA can not, therefore $\boxed{D}$.

30. Use diagram one to find his distance away originally. $d_f - d_o = \boxed{1040.62 = \cancel{X}}$ E

$\tan(76^\circ) = \frac{494}{d_o}$



$\tan(23^\circ) = \frac{494}{d_f}$