FEBRUARY REGIONAL

GEOMETRY

Individual

1. D
2. D
3. B

4. A
5. B
6. A

7. D
8. E
9. C

10. C
11. B
12. B

13. E
14. C
15. E

16. D
17. B
18. C

19. C
20. B
21. A

22. D
23. A
24. B

25. D
26. A
27. B

28. B
29. D
30. C

GEOMETRY INDIVIDUAL SOLUTIONS
FEBRUARY 1999 PAGE 1A

1D. Let h = the thickness of each pizza. Using the formula for volume of a cylinder $V = \pi r^2 h$, we have the proportion $\frac{\$6}{\pi 6^2 h} = \frac{\$x}{\pi\left(7\frac{1}{2}\right)^2 h}$, so $x = \$9.375$, or $9.50 to the nearest half dollar.

2D. Note that the region is not a triangle but instead consists of a triangle and an isosceles trapezoid.

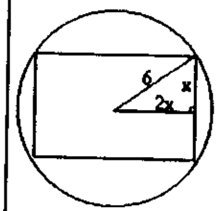
The area of the triangle is $\frac{1}{2}$. The area of the trapezoid is found by $A = \frac{b_1 + b_2}{2}h$, where the bases are 1 and 4 and $h = 4$. The area of the trapezoid is 10. So total area of region is $10\frac{1}{2}$.

3B. The slopes of the two lines are $2k$ and $-\frac{k}{8}$. For the lines to be perpendicular, either the product of the slopes must be −1, or one line is horizontal and the other vertical. The latter case is not possible because no value of k makes either slope undefined.)

Solving $2k\left(-\frac{k}{8}\right) = -1$, we get $k = 2$ or -2 and the sum of these numbers is 0.

4A. Let a = the measure of the angle. Then $\frac{a}{180-a} = \frac{3}{7}$ and $a = 54$. So the angle is 54° and its complement is 36° and the ratio is 3 : 2.

5B. In the figure below, by the Pythagorean theorem, $x = \frac{6}{5}\sqrt{5}$, so the width of the rectangle is $\frac{12}{5}\sqrt{5}$ and the length is $\frac{24}{5}\sqrt{5}$. Thus the area is 57.6.



6A. Since $\Delta ADB \sim \Delta ACE$, $\frac{6}{18} = \frac{4}{6 + BC}$. Thus $BC = 6$.

7D. The desired point is the center of the inscribed circle, which by a theorem is the incenter, the point at which the angle bisectors meet.

8E. For any convex polygon, the sum of the exterior angles, one at each vertex, is 360°, so NOTA.

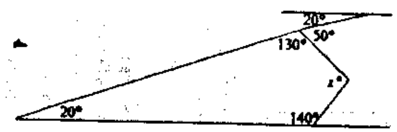
9C. The measure of arc CM = 108° because it is supplementary to $\angle T$. $m\angle U = \frac{1}{2}(108° - 22°) = 43°$

10C. Let the lengths of the segments of one of the chords be $3x$ and $2x$. Then $(3x)(2x) = 16\times 6$, so $x = 4$. Thus the chords have lengths 22 (given) and 20, and the sum is 42.

11B. Using $N = \frac{n(n-3)}{2}$. If $N = 20$, then $n = 8$.

GEOMETRY INDIVIDUAL SOLUTIONS
FEBRUARY 1999 PAGE 1B

12B. Extend one of the segments as shown below, creating a quadrilateral, the sum of whose angles must be 360°. Therefore, $x = 70°$.

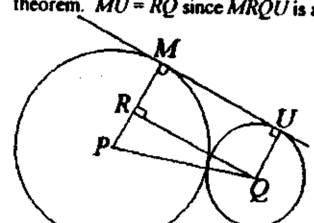


13E. The scarecrow's theorem implies that, with sides a, b, and c, $\sqrt{a} + \sqrt{b} = \sqrt{c}$ even if a is the length of the longes side. But $\sqrt{a} + \sqrt{b} > \sqrt{a} \geq \sqrt{c}$, since a is the length of the longest side. Therefore, NOTA.

14C. The ratio of volumes is 1 : 8. The ratio of the scale factors is the cube root of 1 : 8, or 1 : 2. The ratio of surface areas is the square of the ratio of the scale factors, or 1 : 4.

15E. The quadrilateral could be, for example, a kite, which is none of the choices. Therefore, NOTA.

16D. In the drawing below, $MR = 4$ since $MRQU$ is a rectangle. Therefore $PR = 6$. $PQ = 14$, the sum of the radii. In ΔPRQ, $RQ = 4\sqrt{10}$ by the Pythagorean theorem. $MU = RQ$ since $MRQU$ is a rectangle.



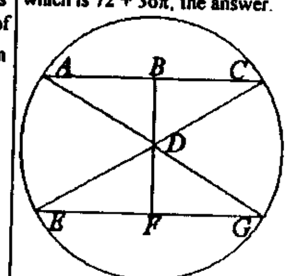
17B. Since $C = 2\pi r$, the radius increased from $\frac{20}{2\pi}$ to $\frac{25}{2\pi}$, a difference of $\frac{5}{2\pi}$.

18C. Since $\overline{AC}$ and $\overline{BD}$, the diagonals of $ABCD$ bisect each other, $ABCD$ is a parallelogram. But since additionally $\overline{AC} \perp \overline{BD}$, $ABCD$ is a rhombus. The figure would only be a square if its diagonals were congruent, that is, if the height and median of the trapezoid were congruent. Therefore, $ABCD$ is a rhombus, the answer.

19C. The diagonals of the rhombus create four 30-60-90 triangles, each with hypotenuse 8. So the triangles have legs 4 and $4\sqrt{3}$. Therefore the shorter diagonal has length 8 and the longer diagonal has length $8\sqrt{3}$. The sum is $8 + 8\sqrt{3}$, the answer.

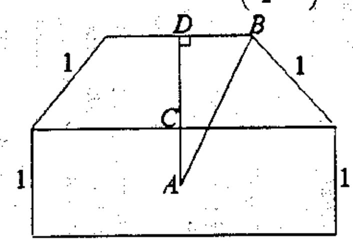
20B. The median of the trapezoid is AB and has length 5, by the distance formula. The area of a trapezoid is equal to the median times the height, and thus is 30, the answer.

21A. In the drawing below, $BC = BD = 6$ and ΔCBD has area 6. ΔABD, ΔEFD, and ΔGFD also have area 6. The area of sector CDG is $\frac{90}{360}\pi\left(6\sqrt{2}\right)^2$ and the area of sector ADE is the same. The area of the shaded region is the sum of the areas of the four triangles and the two sectors, which is $72 + 36\pi$, the answer.



22D. Solving $\frac{360}{n} = \left(\frac{1}{5}\right)\frac{180(n-2)}{n}$, we find $n = 12$, the answer.

23A. In the partial drawing below, $BD = \frac{1}{2}$, $AC = \frac{1}{2}$, and $CD = \frac{\sqrt{2}}{2}$. In ΔADB, by the Pythagorean theorem, $(AB)^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2} + \frac{\sqrt{2}}{2}\right)^2$. Thus $(AB)^2$, which is the radius squared, is $\frac{\sqrt{2}}{2} + 1$, so the area of the circle is $\pi\left(\frac{\sqrt{2}}{2} + 1\right)$, the answer.



24B. The triangle is a right triangle. Since the midpoint of the hypotenuse is equidistant from all three vertices, the median has length 2, the answer.

25D. If the diameter is increased by 1%, the radius is increased by 1% also. Let the original radius be r and the new radius $1.01r$. Then the original area is πr^2 and the new area is $\pi(1.01r)^2$, which is $\pi 1.0201r^2$. The difference in the two areas is $\pi 0.0201r^2$, which if divided by the original area gives 0.0201, which is the same as 2.01%, the answer.

26A. The area of the triangle is 6. Substituting in the formula $A = \frac{1}{2}bh$, we find that for the base with length 5 the height (or altitude) is 2.4. Since this is a right triangle, the legs, which have length 3 and 4, are the other two altitudes. The sum of the lengths of the three altitudes is $3 + 4 + 2.4 = 9.4$, the answer.

27B. By a theorem, the sum of $m\angle G$ and m arc HS is 180°, so $m\angle G = 70°$, the answer.

28B. The area of the sidewalk is the difference between the area of the outer rectangle and the inner rectangle. The area of the outer rectangle is 108 ft by 68 ft, or 7344 sq ft. The area of the inner rectangle is 100 ft by 60 ft, or 6000. The difference is 1344 sq ft, the answer.

29D. The sum of the lengths of all the unlabeled vertical segments must be 24. The sum of the lengths of all the unlabeled horizontal segments must be 20. Therefore the perimeter must be $24 + 24 + 20 + 20 = 88$, the answer.

30C. By Stewart's Theorem, $(AB)^2 \bullet CD + (AC)^2 \bullet BD = BC((AD)^2 + BD \bullet DC)$. Thus $AD = 5$, the answer (An alternate method not requiring this theorem involves drawing a perpendicular from A to $\overline{BD}$ and showing the perpendicular intersects $\overline{BD}$ 1 unit from B. Thus there are two congruent right triangles and $AD = BD$.)