FAMAT February Regional Calculus Team 2005

1. Let A = the length of the longest ladder that can be carried through a 90 degree turn in a corridor 12 meters across. Let B = the length of the shortest ladder that can be put over a fence 6 meters tall and on a wall 6 meters away from it. Find A/B.

2. Let $f(x) = g(x^2) = h(x^3)$. Find $\frac{df}{dg} + \frac{df}{dh} + \frac{dh}{dg}$ at x=1.

3. Let $f(x) = x^3 + x$, $g(x) = f^{-1}(x)$, and $h^{-1}(x) = f(x) + x$. Find $f'(2) + g'(\frac{5}{8}) + h'(\frac{9}{8})$.

4. Peter and Yan are 1000 meters apart. Peter runs towards Yan at 10 meters/sec. Yan spends twice as much time running away from Peter as running towards him, and both are done at constant speed. They meet after 200 seconds. Let A be Yan's speed in meters/sec, let B be the distance in meters they meet from where Yan started, and let C be the total distance in meters they both ran. Find (B+C)/A.

5. A car drives along a flat road directly towards a (very high) mountain. In your view, the mountain is $\frac{\pi}{6}$ radians above the horizon, and this number is presently increasing by 3 radian per hour. Given that the car is traveling at 65mph, how much higher than you is the mountain? Please give your answer in miles.

6. A 10 meter ladder stands vertically next to a wall. Its height changes at dh/dt = -1 meter/second. At what rate, in m^2/sec, does the area it encompasses change when h=5 meters?

7. Let $j(x)$ be continuous on [-4, 11], $\int_{-4}^{6} j(x)dx = p$, and $\int_{6}^{11} j(x)dx = q$. If $\int_{11}^{-4} j(x)+4dx$ is written in the form $Ap+Bq+C$, find $A+B+C$. A, B, and C are real.

8. Consider a pyramid of cheese the base of which is a 10x10 unit square and the height of which is 20 units. A team of mice eats it from the base up, always keeping the same shape. If team A eats it, the rate of change of the height with respect to time reaches 1 unit/minute when the height goes down to 12 units. If team B eats it, the rate of change of the height with respect to time reaches 1 unit/minute when the height goes down to 16 units. Both teams eat at a constant rate. If their eating rates are additive, how long does it take the two teams working together to eat the entire pyramid?

9. A= the four-millionth derivative of $\sin x$ with respect to x, evaluated at x = 0

$$B=\int_{1}^{e} \frac{\ln(x)}{2x} + \frac{\ln(x)^3}{6x} + \frac{\ln(x)^5}{10x} + \frac{\ln(x)^7}{14x} + \ldots + \frac{\ln(x)^{99}}{198x}\,dx$$

Find $A+2B$.

10. The functions $f(x)$ and $g(x)$ intersect at $x=1$, $x=-1$, and one other point. $f(1)=2$, and $f(0)=0$. For all x within [0, 1], $f(1-x)=\frac{g(x-1)}{2}$, and $f(x-1)=2g(1-x)$. $\int_0^1 f(x)\,dx=2$ and $\int_{-1}^1 g(x)\,dx=5$.
What is the area enclosed by $f(x)$ and $g(x)$?

11. Let $A(x)=\int_0^{x^2}\cos(t^2)\,dt$

Let $B(x)=\int_0^{x^2} t^t\,dt$ Find $A'(1)+B'(2)$, rounded to three decimal places.

12. A kite is a planar convex quadrilateral where two adjacent side pairs have equal length, and the diagonals of which intersect at a right angle. Given a kite such that the angle between one of the long sides and one of the short sides is 120 degrees, and the perimeter of which is 20, find the maximum possible area.

13. Let $f(x)=x^3-x+1$. Find the values of $f(x)$ on the interval [-2, 2] which satisfy the Mean Value Theorem. What is the sum of all these values?

14. Suppose I have a sphere with radius R. Out of it, I cut out a cone with the maximum possible volume. Out of that cone, I cut out a cylinder with the maximum possible volume. What is the ratio of the volume of the cylinder to the volume of the cone?

15. What is the resultant volume if you revolve the region bounded by $y=e^x$, x=0, x=1, and y=0 around the x-axis?