Team Solutions – Calculus, February Regional 2005

1. By symmetry, the ladder in A has the most constraints when it's at 45 degrees to each corridor. So the longest length would be A=$24\sqrt{2}$. Similarly for B, the shortest length would be at 45 degrees, so B=$12\sqrt{2}$, and A/B=2

2. $f'(x) = g'(x^2)2x$ => df/dg = 2. Similarly, df/dh=3 and dh/dg = 2/3. So the sum is 17/3.

3. $f'(x) = 3x^2 + 1, g'(\frac{5}{8}) = \dfrac{1}{3(f^{-1}(\frac{5}{8}))^2 + 1}, h'(\frac{9}{8}) = \dfrac{1}{3(inverse\_of(f+x)at(\frac{9}{8}))^2 + 2}$ =>

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4. Each second, they get 5m closer. So Yan must be running away at a net 5m/sec. Since he spends twice as much time running away, his speed is 15m/sec=A. B = (5m/sec)(200sec)=1000m. C = (10m/sec+15m/sec)(200sec)=5000m. (B+C)/A = 400

5. Let a be the height of the mountain and b be the distance from the car to the mountain.

$\tan\theta = \frac{a}{b}$. $\sec^2\theta \frac{d\theta}{dt} = \frac{-a}{b^2}\frac{db}{dt}$, since a is constant. $b^2 = \dfrac{-\frac{db}{dt}(b\tan\theta)}{\frac{d\theta}{dt}\sec^2\theta}$. Plugging in

$\theta = \frac{\pi}{6}$, $\frac{d\theta}{dt} = 3$ and $\frac{db}{dt} = -65$, we get $b = \frac{65\sqrt{3}}{12}$ and a = 65/12

6. $h^2+b^2=100$. So $hdh = -bdb$. $dA = \dfrac{hdb + bdh}{2} = \dfrac{\frac{5}{\sqrt{3}} - 5\sqrt{3}}{2} = \dfrac{-5\sqrt{3}}{3}$

7. We have $\int_{11}^{-4} j(x)dx + \int_{11}^{-4} 4dx = -p - q - 60$, so our total is *-62*

8. Let h be the height of the cheese section that the mice have eaten, and R be the side length of the base. The remaining volume is then $V = \frac{1}{3}R^2(20-h)$. $R = \frac{10(20-h)}{20}$, so $V = \frac{1}{12}(20-h)^3$. $\frac{dV}{dt} = \frac{-1}{4}(20-h)^2\frac{dh}{dt}$. Plugging in h=8 and h=4, we get that the two teams have $\frac{dV}{dt} = 36$ and $\frac{dV}{dt} = 64$. Since the total volume is $V = \frac{1}{12}(20)^3$, the time it takes to eat the entire pyramid is $V = \dfrac{\frac{1}{12}(20)^3}{100} = \dfrac{20}{3}$ minutes.

9. Part A is zero, since the four-millionth derivative is sinx, evaluated at zero is 0. For part B, we can simplify this down to $25\int_1^e \frac{\ln x}{x}dx$, which is 12.5. So when you add the two, we get *12.5.*

10. $g(1) = f(1) = \frac{g(-1)}{2} = \frac{f(-1)}{2}$, and f(0)=g(0)=0.

$\int_0^1 f(x)dx = 2 \Rightarrow \int_{-1}^0 g(x)dx = 4 \Rightarrow \int_0^1 g(x)dx = 1 \Rightarrow \int_{-1}^0 f(x) = 2$. The total area is therefore (4-2)+(2-1)=3

11. $A' = 2x\cos x^4,\ x=1 \Rightarrow 2\cos 1$. $B' = 2x\left(x^2\right)^{x^2} = 4(4^4) = 1024$. The sum is 1025.081

12. Dividing the area into two equal halves through the longer diagonal, we have that the area of each triangle is $\frac{ab\sin(120)}{2}$, where a and b are the side lengths. Since a+b=10, we get that A=$\frac{a(10-a)\sin(120)}{2}$, minimizing which leads to a=5. Since the area of the kite is twice that, we get $\frac{25\sqrt{3}}{2}$.

13. f(-2) = -9, f(2) = 7. So slope = 4. $3x^2$-1=4 => $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$. Since all the parts of the function involving an x are odd, we just get 1(2)=2

14. Let the origin of the sphere be at (0,0). Then the center of the base of the cone is at (0, -h). V = $\frac{\pi(R^2-h^2)(R+h)}{3}$ => dV = (1/3)($R^2 - h^2 - 2h(R+h)$)=0;

$3h^2 + 2hR - R^2 = 0$, so h = R/3, and the volume is $\frac{32\pi R^3}{81}$. Given a height b, the radius c of the cylinder within the cone is $c = \frac{\left(\frac{2\sqrt{2}R}{3}\right)}{\left(\frac{4R}{3}\right)}\left(\frac{4R}{3} - b\right) = \frac{\sqrt{2}\left(\frac{4R}{3} - b\right)}{2}$.

$V = \pi c^2 b = \frac{\pi}{2}\left(\frac{16R^2 b}{9} - \frac{8Rb^2}{3} + b^3\right) \Rightarrow dV = \frac{\pi}{2}\left(\frac{16R^2}{9} - \frac{16Rb}{9} + 3b^2\right)$=0. Solving this, we get b = 4R/9, so the volume of the cylinder is $\frac{128\pi R^3}{729}$. The ratio is therefore 4/9

15. Splitting it up into disks, we get $\pi\int_0^1 \left(e^x\right)^2 = \frac{e^{2x}}{2}\Big|_0^1 = \frac{\pi(e^2-1)}{2}$