FEBRUARY REGIONAL: CALCULUS TEAM

1.

$$A = \lim_{x \to \infty} \frac{6x^2 - 4x - 3}{\sqrt{4x^4 + 4}}$$

$$B = \lim_{x \to 2} \frac{x^3 - 4x}{2x - 4}$$

$$C = \lim_{x \to 0} (e^x - 1)^x$$

Find $A + B + C$.

2. Take the two functions f, g such that $f(x) = x^3 - 12x + 10$ and $g(x) = x^4 - 4x^3 + 4x^2 + 1$. Define A as the sum of the y-values of both relative extremes of f, and B as the sum of the x-values of the three relative extremes of g.

   What is $A + B$?

3. Given the function $f(x) = 2x^4 - 3x^2 - 20$ we can define the following intervals.
   $A :=$ the interval over which the function is negative
   $B :=$ the interval over which the function is increasing
   $C :=$ the interval over which the function is concave up

   How many distinct intervals are contained in $A \cap B \cap C$?

4. A particle moves with a positional equation $x(t) = \cos(2\pi t) + 2t$ over $0 \leq t \leq 4$. How many times over the interval is the particle at rest?

5. The following integral equals $\left(\frac{\pi}{2}\right)^n$ for some value of n, what is that value?

$$\int_0^{\sqrt[3]{\pi/2}} 3x^4 \cos x^3 + 2x \sin x^3 dx$$

6. Consider the function $f(x) = -e^x + c$ for some real number c. For c large enough, the graph passes through the 1st quadrant.

   For all values of c in which the graph passes through the 1st quadrant, find the value of c where the area bounded by the graph and the x and y axes is equal to the value of c. Round your answer to the nearest tenth.

7. Consider the region between the graph of $y = (x-2)^2+3$ and the lines $y = 2$, $x = 1$, and $x = 3$. What is the volume of the solid formed by revolving this region about the line $y = -1$?

8. A rectangle's length is expanding twice as fast as it's width. At the moment when the rectangle's length and width are 3 and 4 respectively, how many times faster than the width is the area expanding?

9. $\mathcal{R}$:= the larger region bounded by the graph of $f(x) = -x^2 + 4x$, the x-axis, and the line $x = 3$.
   A := the area of $\mathcal{R}$.
   V := the volume of the solid formed rotating $\mathcal{R}$ around the y-axis.

   What is the value of AV^{-1} ?

10. Find the derivative with respect to x of the following function at $x = 1$.

$$F(x) = \int_1^{1/x} \frac{e^{xt}}{t}\, dt$$

11. A man stands 30 m away from a target, and fires an arrow horizontally at an initial speed $10\sqrt{30}\ m/s$. How far in meters (exactly) has the arrow fallen vertically when it strikes the target. [Assume the acceleration due to gravity is 10 m/s^2.]

12. A solid S has for its base the region in the xy-plane bounded by the graph of a circle of radius 3. Every cross section in the yz-plane is a square. Find the volume of S.

13. Identify the name of the following theorem.

    "Suppose that (i) $f(x) \leq g(x) \leq h(x)$ for all x near a, and (ii) $\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L$. Then it follows that $\lim_{x\to a} g(x) = L$."

14. For any given function $f(t)$, evaluate the following limit:

$$\lim_{s\to\infty} \int_0^{\infty} e^{-st} f(t)dt =$$

15.

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx =$$