FEBRUARY REGIONAL: CALCULUS TEAM, SOLUTIONS

1. $A = \frac{6}{\sqrt{4}} = 3$; $B = \lim_{x\to 2} = \frac{3x^2-4}{2} = 4$; $\ln C = \lim_{x\to 0} x \ln\left(\frac{e^x}{1}\right) = 0$ $\Rightarrow C = 1 \Rightarrow A + B + C = 8$
2. $f'(x) = 3x^2 - 12x = 0 \Rightarrow x = \pm 2 \Rightarrow A = 26 + (-6) = 20$; $g'(x) = 4x^3 - 12x^2 + 8x = 0 \Rightarrow x = 0, 1, 2 \Rightarrow B = 0 + 1 + 2 = 3$ Therefore $A + B = 20 + 3 = 23$
3. $f(x) = 0 \Rightarrow A = (-2, 2)$; $f'(x) = 8x^3 - 6x = 0 \Rightarrow B = \left(\frac{-\sqrt{3}}{2}, 0\right) \cup \left(\frac{\sqrt{3}}{2}, \infty\right)$; $f''(x) = 24x^2 - 6 = 0 \Rightarrow C = (-\infty, -1/2) \cup (1/2, \infty)$ $\Rightarrow A \cap B \cap C = \left(\frac{-\sqrt{3}}{2}, -1/2\right) \cup \left(\frac{\sqrt{3}}{2}, 2\right)$. Therefore there are 2 distinct intervals
4. Particle at rest when velocity is zero. $v(t) = x'(t) = 2 - 2\pi \sin(2\pi t) = 0 \Rightarrow 8$ roots.
5. $\int_0^{\sqrt[3]{\pi/2}} 3x^4 \cos x^3 + 2x \sin x^3 dx = x^2 \sin x^3\Big]_0^{\sqrt[3]{\pi/2}} = \frac{\pi}{2}^{2/3} \Rightarrow n = \frac{2}{3}$
6. $c = \int_0^{\ln c}(-e^x + c)dx \Rightarrow c = c \ln c - c + 1 \Rightarrow c \approx 6.3$
7. $V = \pi \int_1^3 [((x-2)^2 + 4)^2 - 3^2]dx = \frac{296\pi}{15}$
8. $\frac{dl}{dt} = 2\frac{dw}{dt}$. $A = lw \Rightarrow \frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt} = l\frac{dw}{dt} + 2w\frac{dw}{dt} \Rightarrow \frac{dA}{dt} = (3 + 2\cdot 4)\frac{dw}{dt} = 11\frac{dw}{dt}$
9. $A = \int_0^3 -x^2 + 4x\ dx = 9$. $V = 2\pi \int_0^3 x(-x^2 + 4x)\ dx = \frac{63\pi}{2}$. Therefore $AV^{-1} = 9 \cdot \frac{2}{63\pi} = \frac{2}{7\pi}$
10. $F'(x) = \frac{e^{x(1/x)}}{1/x}\left(-\frac{1}{x^2}\right) - e^x(0) + \int_1^{1/x} e^{xt}\ dt = -\frac{e}{x} + \left[\frac{e^{xt}}{x}\right]_1^{1/x} = -\frac{e}{x} + \frac{e}{x} - \frac{e^x}{x} = -\frac{e^x}{x} \Rightarrow$ $F'(1) = -e$
11. $x = vt \Rightarrow t = \frac{30}{10\sqrt{30}} = \frac{\sqrt{30}}{10}$. $-y = \frac{1}{2}gt^2 = -5\left(\frac{\sqrt{30}}{10}\right)^2 = \frac{3}{2}$
12. $V = \int A_{cross}\ dx = \int_{-3}^3 (2\sqrt{9 - x^2})^2\ dx = 144$
13. Squeeze(ing) Theorem or Sandwich Theorem
14. As $s \to \infty$: $\int_0^\infty e^{-st} f(t)dt \longrightarrow 0$
15. $\int_{-\infty}^\infty e^{-x^2} dx = \sqrt{\pi}$. (Gauss's integral)

Calculus Individual Answers

1. B
2. C
3. D
4. A
5. C
6. A
7. B
8. A
9. B
10. D
11. B
12. C
13. C
14. A
15. B
16. B
17. A
18. E
19. C
20. C
21. D
22. A
23. B
24. C
25. B
26. D
27. D
28. C
29. A
30. D

Calculus Team Answers

1. 8
2. 23
3. 2
4. 8
5. $\frac{2}{3}$
6. 6.3
7. $\frac{296\pi}{15}$
8. 11
9. $\frac{2}{7\pi}$
10. $-e$
11. $\frac{3}{2}$
12. 144
13. Squeeze(ing) Theorem or Sandwich Theorem
14. 0
15. $\sqrt{\pi}$