February 2000 FAMAT Regional Mathematics Competition
Calculus Team Test

1. A = the limit as x approaches 1 of $\frac{x^2-1}{x^2+6x-7}$

   B = the limit as x approaches $+\infty$ of $\frac{2(x-4)+2x-4}{\sqrt{x^2-8x+3}}$

   C = the limit as x approaches $-\infty$ of $\frac{3^x-3^{-x}}{3^x+3^{-x}}$

   D = the limit as x approaches $+\infty$ of $\left(\frac{\ln x}{x}-\frac{1}{\sqrt{x}}\right)$

   Find the value of ABCD.

2. A is the equation $y = x^2$ B is the equation $x = \frac{1}{27}y^2$
   C is the equation $x = 1$ D is the equation $y = 6$.
   Let X be the area of the region bounded by equations A and B.
   Let Y be the area of the region bounded by equations A and B and lying to the right of equation C.
   Let Z be the area of the region bounded by equations A, B, C and D.
   Find the value of $X - Y + Z$.

3. A = the maximum value of the function $y = -2x^2 + 4x - 1$
   B = the minimum value of the function $y = x^2 + 6x - 1$
   C = the ordinate of the inflection point of the graph $y = x^3 + 6x + 2$
   D = the radius of the circle described by the equation $r = 6\cos\theta$
   Find the value of $\text{A}+\text{B}+\text{C}+\text{D}$.

4. A solid metal sheet at a temperature of 40 degrees Fahrenheit is completely submerged in a large container of water that is maintained at a constant temperature of 200 degrees Fahrenheit. Ten minutes later, the sheet is at a temperature of 120 degrees. The rate of change of the temperature of the metal sheet is proportional to the difference between the temperature of the metal sheet and the temperature of the water.
   Find the positive difference (in degrees Fahrenheit) between the temperature of the sheet 20 minutes after and 30 minutes after it is placed in the water.

5. $\text{A} = f(1)$ $\text{B} = f'(1)$ $\text{C} = f''(1)$ $\text{D} = f'''(1)$

   If $f(x) = \int_1^{x^2-1} (4a^3 - 2a)\,da$, find $A + B + C + D$.

6. A solid is formed so that every cross section perpendicular to the x-axis is an equilateral triangle with two vertices on the circle $x^2 + y^2 = 9$. Find the volume of revolving this solid about the line x = 5.

7. Sand pours out of an inverted conical container at a rate of 4 cubic inches per second. By means of a divider, the sand is split so that half goes to side A and half goes to side B. On side A, the sand falls on to the ground and forms a conical pile such that the height of the new pile is always half the radius of the base of the cone. On side B, there is a lot more humidity and so the sand falls on to the ground and forms a conical pile such that the height of the new pile is always twice the radius of the base of the cone. Find the ratio of the rate at which the height of the cone on side A is changing to the rate at which the height of the cone on side B is changing when the cone on side A has a base area of 36π.

8. $y^2 = x^3 - x$ $\qquad$ $ab = a^3 - b^2$

$G = \dfrac{d^2y}{dx^2}$ evaluated at $(2, \sqrt{6})$ $\qquad$ $H = \dfrac{d^2b}{da^2}$ evaluated at $a = 2$ & $b = 2$

Find the value of GH.

9. Using $n = 4$ for both approximations, find the positive difference between approximating $\int_0^1 \sqrt{1+x}\,dx$ with the trapezoidal rule and with Simpson's rule. Round your final answer to the nearest thousandth.

10. A = the volume of a solid that has a base in the shape of an ellipse with a major axis of length 10 and a minor axis of length 8. Every cross section perpendicular to the major axis is an isosceles right triangle with one leg in the plane of the base.
B = the volume of a right circular cone with a height of 20. The base of the cone is an ellipse with a major axis of length 16 and a minor axis of length 8.
Find B/A.

11. A = the radius of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{2\sqrt{n}}$

B = the limit as n approaches infinity of $\displaystyle\sum_{x=1}^{n} \left(\frac{(x+2n)^2}{4n^3} \right)$

C = $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k(k+1)(k+2)}$

Find the value of ABC.

12. $A = \displaystyle\int_0^1 \frac{Arc\tan x}{1+x^2}dx$ $\qquad$ $Arc\sin(Gx + H) + C = \displaystyle\int \frac{dx}{\sqrt{6+x-x^2}}$

Find the value of AGH.

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13. A = the limit as x approaches 1 of $\frac{1}{\ln x}-\frac{x}{x-1}$

B = the limit as x approaches 0 of $(\cos x)^{1/x}$

C = the limit as x approaches $\frac{\pi}{2}$ of $(\sin x-\cos x)^{\tan x}$.

Find ABC.

14. The faces of two identical tetrahedrons are joined to form a solid with six identical faces. The volume of this solid is $36\sqrt{2}$. Find the length of a side of one of the original tetrahedrons in order to minimize the surface area of the final solid.

15. $y=3^{2x+3^{2x+3^{\cdot^{\cdot^{\cdot}}}}}$. Evaluate dy/dx at $x=-1$.