Calculus Team Solutions Feb. 8, 1991

1. $f'(x) = [-\sin x + \cos x - \sec^2 x]$

$f'\left(\frac{2\pi}{6}\right) = -\sin\frac{2\pi}{6} + \cos\frac{2\pi}{6} - \left(\sec\frac{2\pi}{6}\right)^2 = -\left(-\frac{1}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right) - \frac{4}{3}$

$= \boxed{\frac{-5-3\sqrt{3}}{6}}$

2. $\lim_{x\to 0} \frac{1-\cos x}{x} = 0 = A$ $\lim_{x\to 7} \frac{x^2-2x-35}{x^2-49} = \lim_{x\to 7} \frac{x+5}{x+7} = \frac{6}{7} = B$

$\lim_{x\to -4} \sqrt{x^2+9} = 5 = C$ $\frac{7B+5A}{C} = \frac{7\left(\frac{6}{7}\right)+5(0)}{5} = \boxed{\frac{6}{5}}$

3. $x = 2$ $A = 2$

$f'(x) = \frac{x^2-4x}{(x-2)^2} = 0$

$x(x-4) = 0$

$x = 0, 4$

$B = 4$

$y = \frac{0-2(0)+4}{0-2} = -2 = C$

$\frac{A}{C} + B = \frac{2}{-2} + 4 = \boxed{3}$

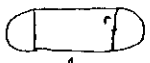
4. $\frac{1}{2-0}\int_0^2 x(x^2+1)^3\,dx = \frac{1}{2}\int \frac{1}{2}u^3\,du = \frac{1}{4}\cdot\frac{u^4}{4} = \frac{1}{16}(x^2+1)^4\Big|_0^2$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{16}[625-1] = \frac{624}{16} = \boxed{39}$

5. 

$A = 2lr + \pi r^2$

$P = 2\pi r + 2l = 300$

$l = 150 - \pi r$

$A = 2r(150-\pi r) + \pi r^2$

$A = 300r - 2\pi r^2 + \pi r^2$

$A = 300r - \pi r^2$

$A' = 300 - 2\pi r = 0$

$r = \boxed{\frac{150}{\pi}}$

$A'' = -2\pi < 0$, max

6. [illegible]

7. $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \frac{4h^2}{9}\cdot h$

$V = \frac{4}{27}\pi h^3$

$\frac{r}{h} = \frac{4}{6}$

$r = \frac{2h}{3}$

$\frac{dV}{dt} = \frac{4}{9}\pi h^2 \frac{dh}{dt} \rightarrow 12 = \frac{4\pi}{9}(9)\frac{dh}{dt} \rightarrow \frac{dh}{dt} = \boxed{\frac{3}{\pi}}$

8. $\int \text{Arctan}\, x\,dx = x\,\text{Arctan}\, x - \int \frac{x}{1+x^2}\,dx$

$u = \text{Arctan}\, x \quad dv = dx$

$du = \frac{1}{1+x^2}dx \quad v = x$

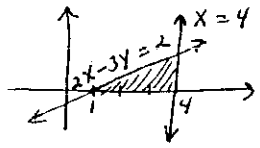
$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| = \frac{1}{2}\ln(1+x^2)$

$\int \text{Arctan}\, x\,dx = \boxed{x\,\text{Arctan}\, x - \frac{1}{2}\ln(1+x^2) + C}$

9.



$2x - 3y = 2$

$y = \frac{2}{3}x - \frac{2}{3}$

$A = 2\pi \int_1^4 x\left(\frac{2}{3}x - \frac{2}{3}\right)dx =$

$2\pi \cdot \frac{2}{3}\left[\frac{x^3}{3} - \frac{x^2}{2}\right]\Big|_1^4 = \frac{4\pi}{3}\left[21 - 8 + \frac{1}{2}\right]$

$= \frac{4\pi}{3}\left(\frac{27}{2}\right) = \boxed{18\pi}$

10. $x^2 + 2xy + y = 3$, $x = -1$, $y =$

$2x\frac{dx}{dt} + 2y\frac{dx}{dt} + 2x\frac{dy}{dt} + \frac{dy}{dt} = 0$

$2(-1)(4) + 2(-2)(4) + 2(-1)\frac{dy}{dt} + \frac{dy}{dt} = 0$

$-8 - 16 - \frac{dy}{dt} = 0$

$\frac{dy}{dt} = \boxed{-24}$

11. $\lim_{t\to 0} \frac{(t+2)^7 - 2^7}{t} = \lim_{t\to 0} \frac{t^7 + 7t^6\cdot 2 + 21t^5\cdot 4 + \cdots + 2^7 - 2^7}{t} =$

$\lim_{t\to 0}\left(t^6 + 7t^5\cdot 2 + 21t^4\cdot 4 + \cdots + 7\cdot 2^6\right) = \boxed{448}$

12. $f(x) = x^3 - 3x^2 - 6x + 8$, roots are $4, 1, -2$, $A = 3$

$f'(x) = 3x^2 - 6x - 6$, roots are $1 \pm \sqrt{3}$, $B = 2$

$f''(x) = 6x - 6$, root is 1, $C = 1$ $\quad \frac{B+C}{A} = \frac{2+1}{3} = \boxed{1}$

13. $V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$-16\pi = 4\pi r^2(-4)$

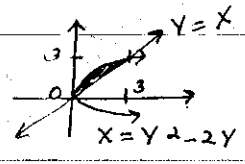
$1 = r^2$

$\boxed{1} = r$

14. $Y = \frac{5^{2x}}{\ln 5}$ $\quad Y' = \frac{5^{2x}\cdot 2\cdot \ln 5}{\ln 5} = 2\cdot 5^{2x}$

$= 5^4 \cdot 2 = \boxed{1250}$

15.



$\int_0^3 [y - (y^2 - 2y)]dy = \int_0^3 (3y - y^2)dy$

$= \frac{3}{2}y^2 - \frac{y^3}{3}\Big|_0^3 = \boxed{\frac{9}{2}}$