Note: Throughout the test you may mark "E" for none of these answers.

1. A particle moves at $dy/dt=(t-3)^2$, $dx/dt=(t-3)^2$. How much distance does it travel from t = 0 to t = 9 (in units)?
   A. 63
   B. $63\sqrt{2}$
   C. 81
   D. $81\sqrt{2}$
   E. NOTA

2. Find the intersection point of the tangent line to $y=x^2$ at $x=2$ and the perpendicular to the tangent to $y=x^3$ at $x=1$.
   A. (16/13, 12/13)
   B (4/7, -2/7)
   C. (2, 4)
   D. (4, 10)
   E. NOTA

3. How many total relative minima/maxima and points of inflection does $f(x)=x^4-14x^2+24x$ have?
   A. 2
   B. 3
   C. 4
   D. 5
   E. NOTA

4. The area of a regular hexagon starts infinitesimally small and increases by t cm^2/sec at all times $t>0$. The shape continues to be a regular hexagon. At the time the area is 100 cm^2, how fast is the length of each side changing?
   A. $\frac{10\sqrt{3}}{9}$ cm/se
   B. $\frac{10\sqrt{6\sqrt{3}}}{27}$ cm/sec
   C. $\frac{3\sqrt{2}}{10}$ cm/sec
   D. $\frac{\sqrt{\sqrt{3}}}{3}$ cm/sec
   E. NOTA

5. If $dz/dx = d^2z/dx^2 = 0$ and $2y + x^2 + z = 3xz$, then find d^2y/dx^2.
   A. -1
   B. $-x$
   C. $z - x$
   D. Not enough information
   E. NOTA

6. Integrate $f(x) = \dfrac{3x}{4x^2+1}$ with respect to x from $x = 1$ to 3. Give the hundredth digit of your answer.
   A. 0
   B. 3
   C. 5
   D. 8
   E. NOTA

7. What is the number of solutions to $\int_0^x \sin^2(t^2)\,dt = 0$?
   A. 0
   B. 1
   C. 2
   D. Infinitely many
   E. NOTA

8. Find the 100th derivative with respect to x of $f(x) = \sin x \cos x$ evaluated at $x = \pi/12$.
   A. $2^{97}\pi$
   B. 2^{98}
   C. 2^{99}
   D. $2^{99.5}$
   E. NOTA

9. Evaluate $\int_2^3 \dfrac{dx}{x^3 - x^2}$ to six decimal places.
   A. 0.526481
   B. 0.548612
   C. 0.121019
   D. 0.121020
   E. NOTA

10. Find the domain of $f(x)=\sqrt{\ln\left(\sqrt{\frac{x+2}{x+1}}\right)}$

A. $x\neq -1$
B. $x\neq -2, x\neq -1$
C. $x>-1, x<-2$
D. $x>-1$
E. NOTA

11. $f(b)=\lim_{x\to 0^+}\frac{(b+10^{-2x})^{-4x}-1}{x}$ Find $f'(2)$.

A. 2log3
B. 3ln2
C. ln3 + 2/3
D. log2 + 3/2
E. NOTA

12. $\int_{-\pi}^{\pi}\frac{x^2\sin(x)}{x^4+1}dx$

A. $5/11+2\pi/3$
B. 5/11
C. $2\pi/3$
D. 0
E. NOTA

13. Wire of 0.1cm radius is tightly wound (with no gaps in between layers) around a cylindrical pole of 3cm radius between the heights of 0cm and 1cm. As a result, this part of the pole thickens, and the new radius is 4+h cm, where h is the height. How many centimeters of wire were used up? (Hint: Length * Cross-sectional area = Volume)

A. $\frac{850}{3}$
B. $\frac{850\pi}{3}$
C. $\frac{3400}{3}$
D. $\frac{3400\pi}{3}$
E. NOTA

14. Suppose that f is twice differentiable over all real numbers, and $f(0) = f'(0) = f''(0) = 0$ If $f'(x) \le 2$, $f''(x) \le 2$ for all x, what is the maximum possible value of f on [0, 10]?
   A. 9
   B. 11
   C. 18
   D. 19
   E. NOTA

15. Find the slopes of all lines tangent to both $f(x) = 3x^2 - 4x$ and $g(x) = x^2 - 2$.
   A. 2
   B. 2, -10
   C. $1 \pm \frac{\sqrt{6}}{3}$
   D. 5, -11/3
   E. NOTA

16. There's a small strip of land that runs vertically along $x = 2$, and two islands, one at (0, 0) and one at (5, 5). Gorav swims at 3 units/hr and runs at 10 units/hr. How much distance should he run along the shore to complete the trip between the two islands as fast as possible? Round your answer to the nearest hundredth.
   A. 0
   B. 2.20
   C. 3.10
   D. 3.43
   E. NOTA

17. Let $f(x) = xg(x) - x$. For $n > 1$, what is the n^{th} derivative of $f(x)$ with respect to x?
   A. $ng^{(n)}(x) + g^{(n-1)}{}'(x)$
   B. $xg^{(n)}(x) + ng^{(n-1)}(x)$
   C. $g^{(n)}(x) + ng^{(n-1)}(x)$
   D. $ng^{(n)}(x) + xg^{(n-1)}(x)$
   E. NOTA

18. What is the average value of $f(x) = e^x - e^{2x}$ over $x = [0, 10]$? Please round your answer to three significant figures.
    A. -2.43×10^7
    B. -1.25×10^7
    C. -9.43×10^6
    D. -3.54×10^6
    E. NOTA

19. What is the area between the graphs of $f(x) = \sin x$ and $g(x) = \cos x$ between $x = -3\pi/4$ and $x = \pi/4$?
    A. $\frac{\sqrt{2}}{2}$
    B. $\sqrt{2}$
    C. $2\sqrt{2}$
    D. $4\sqrt{2}$
    E. NOTA

20. $h(x) = \frac{f(x)g(x)}{f(x)+1}$ $f(2) = 1$, $g(2) = 2$, $f'(2) = 3$, $g'(2) = 5$. Find $h'(2)$
    A. 5/2
    B. 3
    C. 7/2
    D. 8
    E. NOTA

21. Differentiate $f(x) = e^{e^{e^x}}$ at $x = 1$ with respect to x. Find the third significant digit of your answer.
    A. 0
    B. 4
    C. 7
    D. 9
    E. NOTA

22. Let $f(x) = (\sec x + \csc x)^2$. Find the 6th derivative of $f(x)$ with respect to x evaluated at $x = \pi$.
    A. -32
    B. 0
    C. 32
    D. undefined
    E. NOTA

23. If $f(y) = \int_{2}^{y-2} 3x^2\,dx$, then what is $\int_{1}^{2} f(y)\,dy$?

A. -9
B. -8.25
C. -8.5
D. -8
E. NOTA

24. What is happening to the graph of $f(x) = \frac{1}{6}x^3 + \ln x - 1$ at $x = 2$?

A. increasing, concave up
B. increasing, concave down
C. decreasing, concave up
D. decreasing, concave down
E. NOTA

25. If $\ln x = e^{f(x)}$, then $f'(e) =$

A. e
B. 1
C. $\frac{1}{e}$
D. -1
E. NOTA

26. For what value of x does $f(x) = 2\ln(x^3 - 1) + x^3 + 2 + \frac{2}{x^3 - 1}$ have a relative maximum?

A. $-\sqrt[3]{2}$
B. -1
C. $\sqrt[3]{2}$
D. The function does not have a relative maximum
E. NOTA

27. If $f(x)=2x-2x^2$, at the instant that x is changing at a rate of 2 $\frac{\text{un}}{\text{sec}}$ and $f(x)$ is changing at a rate of 1.5 $\frac{\text{un}}{\text{sec}}$, then x =

A. 0
B. $-\frac{3}{16}$
C. $-\frac{5}{16}$
D. $-\frac{7}{16}$
E. NOTA

28. Suppose a surface is formed by revolving the graph of $f(x) = x^2$ from $x = 0$ to $x = 1$ around the y-axis. I now fill the solid that this surface bounds with water at the rate $R(t) = t^3$ units^3/sec. At what rate is the depth of the water changing when the container is half-full? Express your answer to the nearest hundredth and in units/sec. Hint: the volume of a paraboloid is (1/2)*(Area of base)*(height).
A. 0.60
B. 1.41
C. 1.42
D. 2.72
E. NOTA

29. Which integral below gives the area between $y = x^4 - x^2$ and $y = |x|$ on the interval $-a \le x \le a$? Let a and $-a$ be the x-coordinates of the points of intersection between the two graphs.

A. $2\int_{-a}^{0} x^4 - x^2 - x dx$

B. $\int_{-a}^{a} x^4 - x^2 - |x| dx$

C. $\int_{0}^{a} (|x| - x^4 + x^2)^2 dx$

D. $2\int_{-a}^{0} -x - x^4 + x^2 dx$

E. NOTA

30. Differentiate $\lim_{n\to\infty} 1 + x + x^2 + x^3 + ...x^n$ at x = ½.

A. 1
B. 2
C. 4
D. 8
E. NOTA