FEBRUARY REGIONAL: CALCULUS INDIVIDUAL TEST, SOLUTIONS

B 1. $f(x) = \ln[(x-3)(x+1)] \cdot \sqrt[3]{(x+5)(x-2)}$; ln positive on $(-\infty, -1) \cup (3, \infty)$

C 2. $\lim_{x\to\infty} \frac{4x^2+3x+2}{\sqrt{4x^4+2}} \longrightarrow \lim_{x\to\infty} \frac{4x^2}{\sqrt{4x^4}} = \lim_{x\to\infty} \frac{4x^2}{2x^2} = 2$

D 3. $\lim_{h\to 0} \frac{2^{2(1+h)+1}-8}{h} = \left(2^{2x+1}\right)'(1) = \left(2^{2x+1} 2\ln 2\right)(1) = 16\ln 2$

A 4. $r(\theta) = \cos^2\theta \Rightarrow r'(\theta) = -2\cos\theta\sin\theta \Rightarrow r'(\frac{\pi}{4}) = -1$

C 5. $f(x) = \frac{\exp(x^2)}{2x} \Rightarrow f'(x) = \exp(x^2)\left(1 - \frac{1}{2x^2}\right) \Rightarrow f''(x) = \exp(x^2)\left(2x - \frac{1}{x} + \frac{1}{x^3}\right) \Rightarrow$ $f''(2) = \frac{29}{8}e^4$

A 6. $y'(x) = f(g(x)) \cdot h'(x) + f'(g(x)) \cdot g'(x) \cdot h(x) \Rightarrow y'(2) = -6 + (-1) = -7$

B 7. $f^{(2001)}(x) = 2001! - 2\sin x - 3\cos x - e^{-x} \Rightarrow f^{(2001)}(0) = 2001! - 3 - 1 = 2001! - 4$

A 8. $f''(x) = 2(f(x))^3$, $f'''(x) = 6(f(x))^4 \Rightarrow f^{(n)}(x) = n!(f(x))^{n+1}$

B 9. $f'(x) = 6x^2 - 6x - 4 = 0 \Rightarrow x = \frac{3\pm\sqrt{33}}{6}$; $f''(x) = 12x - 6 = 0 \Rightarrow x = \frac{1}{2}$; $f' < 0$ for $\left(\frac{3-\sqrt{33}}{6}, \frac{3+\sqrt{33}}{6}\right)$; $f'' < 0$ for $(\frac{1}{2}, \infty) \Rightarrow$ decr. & conc. up on $\left(\frac{1}{2}, \frac{3+\sqrt{33}}{6}\right)$

D 10. $f'(x) = \frac{(1+x^2)^{3/2} - \frac{3}{2}x\sqrt{1+x^2}}{(1+x^2)^3} = 0 \Rightarrow x = \pm\frac{\sqrt{2}}{2}$; $\min = f\left(\frac{-\sqrt{2}}{2}\right) = \frac{-2}{3^{3/2}}$, $\max = f\left(\frac{\sqrt{2}}{2}\right) = \frac{2}{3^{3/2}} \Rightarrow \left[\frac{-2}{3^{3/2}}, \frac{2}{3^{3/2}}\right]$

B 11. $f'(x) = x\sin 2x \Rightarrow h(x) = \frac{4\sin x\cos x}{\cos^2 x} = 4\tan x \Rightarrow h'(x) = 4\sec^2 x$

C 12. I and III are true

C 13. n values of c where $f'(x) = 0$

A 14. $f(0) = 0$; $f'(x) = -\sin\frac{\pi}{2}x - \frac{2}{\pi}e^x \Rightarrow f'(0) = -\frac{2}{\pi}$; $y(x) = -\frac{2}{\pi}x \Rightarrow f(1) \approx y(1) = -\frac{2}{\pi}$

B 15. $f'(x) = 2ax \Rightarrow m_{normal} = \frac{-1}{2ax}$. Two lines are $x = 0$ and $y = \frac{-1}{2a}(x-1) + a$. x-intercept$= 2a^2 + 1$, y-intercept$= a + \frac{1}{2a} \Rightarrow \tan\phi = 2a$

B 16. $x^3 - 2x^2 + 3x - 4 = -2 \Rightarrow x = 1; g'(-2) = \frac{1}{f'(1)} = \frac{1}{3(1)^2 - 4(1) + 3} = \frac{1}{2}$

A 17. Intersect @ (1, 1); $f'(1) = 2$, $g'(1) = 3 \Rightarrow \tan\theta = \frac{3-2}{1+(3)(2)} \Rightarrow \theta \approx 8.130$

E 18. $D = t^3 - t^2; \forall t > 1 \Rightarrow \frac{dD}{dt} = 3t^2 - 2t \Rightarrow \frac{dD}{dt}(2) = 3(2)^2 - 2(2) = 8$

C 19. By similar $\triangle$'s $\frac{h}{y} = \frac{j}{x+y} \Rightarrow hx = (j-h)y$, $h\frac{dx}{dt} = (j-h)\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = \frac{hv}{j-h}$. Rate of tip is $\frac{d}{dt}(x+y) = v + \frac{hv}{j-h} = \frac{vj}{j-h}$

C 20. $f(x+\Delta x) = f(x) + f'(x)(\Delta x) \Rightarrow \sqrt[3]{1320} = \sqrt[3]{1331} + \frac{1}{3}(1331)^{-2/3}(-11) = \frac{362}{33}$

D 21. $x + y = 2\ln(x+y) + 1 \Rightarrow 1 + y' = 2\frac{1+y'}{x+y} \Rightarrow y' = \frac{x+y-2}{2-x-y} = -1$

A 22. Easily obtained $\frac{dy}{dx} = -\frac{x}{y} \Rightarrow \frac{d^2y}{dx^2} = \frac{y(-1)-(-x)y'}{y^2} = \frac{x\left(\frac{-x}{y}\right)-y}{y^2} = \left[-\frac{x^2}{y^3} - \frac{1}{y}\right]$ @$(0, r) = \frac{-1}{r}$

B 23. Substitute $u = \ln x \Rightarrow \int_1^e \frac{\ln x}{x}\,dx = \int_0^1 u\,du = \frac{1}{2}$

C 24. $\int_0^\pi \sin^2 x\,dx = \int_0^\pi \frac{1}{2} - \frac{\cos 2x}{2}\,dx = \left[\frac{x}{2} - \frac{\sin 2x}{4}\right]_0^\pi = \frac{\pi}{2}$

B 25. $\frac{e^x+e^{-x}}{2} = \frac{e+e^{-1}}{2}, x = \pm 1$. $A = \int_{-1}^1 \frac{1}{2e} + \frac{e}{2} - \frac{e^x+e^{-x}}{2}\,dx = \left[\left(\frac{1}{2e} + \frac{e}{2}\right)x - \frac{e^x - e^{-x}}{2}\right]_{-1}^1 = \frac{2}{e}$

D 26. $f(x) = x(x-3)(x-1)$. $A = \int_0^1 (x^3 - 4x^2 + 3)dx + \int_1^3 (-x^3 + 4x^2 - 3)dx = \frac{5}{12} + \frac{8}{3} = \frac{37}{12}$

D 27. Substitute $u = x^2 + 4 \Rightarrow \int_0^{4\sqrt{2}} \frac{2x}{\sqrt{x^2+4}} dx = \int_4^{36} u^{-1/2}\, du = [2\sqrt{u}]_4^{36} = 8$

C 28. $v(t) = 2at + b \Rightarrow \bar{v}_{[d,e]} = \frac{1}{e-d} \int_d^e (2at + b) dt = \frac{1}{e-d} \left[at^2 + bt\right]_d^e = a(e + d) + b$

A 29. $V = 2\pi \int_1^2 x(x^2)\, dx = 2\pi \left[\frac{x^4}{4}\right]_1^2 = \frac{15\pi}{2}$

D 30. By Theorem of Pappus, $V = 2\pi \bar{z} A = 2\pi(2\sqrt{2})(4\pi) = 16\sqrt{2}\pi^2$

Calculus Individual Answers

1. B
2. C
3. D
4. A
5. C
6. A
7. B
8. A
9. B
10. D
11. B
12. C
13. C
14. A
15. B
16. B
17. A
18. E
19. C
20. C
21. D
22. A
23. B
24. C
25. B
26. D
27. D
28. C
29. A
30. D

Calculus Team Answers

1. 8
2. 23
3. 2
4. 8
5. $\frac{2}{3}$
6. 6.3
7. $\frac{296\pi}{15}$
8. 11
9. $\frac{2}{7\pi}$
10. $-e$
11. $\frac{3}{2}$
12. 144
13. Squeeze(ing) Theorem or Sandwich Theorem
14. 0
15. $\sqrt{\pi}$