NOTA—stands for None of the Above

1. A particle moves on the x-axis in such a way that its position at time t, $t > 0$, is given by $x(t) = (\ln t)^2$. At what value of t does the velocity of the particle attain its maximum?

   a) 1 b) $e^{\frac{1}{2}}$ c) e d) $e^{\frac{3}{2}}$ e) NOTA

2. The function f is continuous at the point (c, f (c)). Which of the following statements could be false (if any)?

   a) $\lim_{x\to c} f(x)$ exists b) $\lim_{x\to c} f(x) = f(c)$ c) $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$ d) f(c) is defined e) NOTA

3. Let f and g be functions whose derivatives exist for all real numbers, with $g(x) \neq 0$ for $x \neq 0$. If $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} g(x) = 0$ and $\lim_{x\to 0} f'(x) = 6$ and $\lim_{x\to 0} g'(x) = 2$, then $\lim_{x\to 0} \frac{f(x)}{g(x)}$ is

   a) 0 b) 1 c) 3 d) Nonexistent e) NOTA

4. $\lim_{h\to 0} \frac{\int_1^{1+h} \sqrt{x^3+8}\,dx}{h}$ is

   a) 0 b) 1 c) 3 d) $2\sqrt{2}$ e) NOTA

5. Using the method of partial fractions to decompose $\frac{5x-1}{x^2-1}$, one of the fractions obtained is

   a) $\frac{2}{x+1}$ b) $\frac{3}{x-1}$ c) $\frac{2}{x-1}$ d) $\frac{5}{x+1}$ e) NOTA

6. If $y = \left(\frac{x^3-2}{2x^5-1}\right)^4$, find $\frac{dy}{dx}$ at $x = 1$.

   a) –52 b) –28 c) –13 d) 13 e) NOTA

7. The slope of a line tangent to the graph of $\ln(x + y) = x^2$ at the point where $x = 1$ is

a) 0 b) 1 c) $e - 1$ d) $2e - 1$ e) NOTA

8. If $e^{g(x)} = 2x+1$, then $g'(x) =$

a) $\frac{1}{2x+1}$ b) $\frac{2}{2x+1}$ c) $2(2x+1)$ d) e^{2x+1} e) NOTA

9. The average value of the function $f(x) = (x-1)^2$ from $x = 1$ to $x = 5$ is

a) $\frac{-16}{3}$ b) $\frac{16}{3}$ c) $\frac{64}{3}$ d) $\frac{66}{3}$ e) NOTA

10. If $x = t^2$ and $y = \ln(t^2+1)$, then at $t = 1$, $\frac{d^2y}{dx^2}$ is

a) -¼ b) -½ c) –1 d) 0 e)NOTA

11. If $f(x) = 3x^2 - x$, and $g(x) = f^{-1}(x)$, then $g'(10)$ could be

a) 59 b) $\frac{1}{59}$ c) 11 d) $\frac{1}{11}$ e) NOTA

12. The base of a solid is the region in the first quadrant bounded by the curve $y = \sqrt{\sin x}$ for $0 \le x \le \pi$. If each cross-section of the solid perpendicular to the x-axis is a square, the volume of the solid is

a) 0 cu. units b) 1 cu. units c) 2 cu. units d) 3 cu. units e) NOTA

13. Consider the curve $5x - xy + y^2 = 7$. The slope of the line tangent to the curve at the point (1, 2) is

a) -2 b) –1 c) 0 d) 1 e) NOTA

14. If f is continuous at x = 2, and if $\begin{cases} f(x)=\dfrac{\sqrt{x+2}-\sqrt{2x}}{x-2} & \text{for } x\neq 2 \\ f(2)=k \end{cases}$, then k =

a) – ½ b) – ¼ c) ½ d) ¼ e) NOTA

15. The slope of the tangent line to the curve $y(\cos x)+e^{y}=5$ at the point where $x=\dfrac{\pi}{2}$ is

a) 0 b) 5 c) $\dfrac{\ln 5}{5}$ d) $1+\dfrac{\ln 5}{5}$ e) NOTA

16. If $f(x)=\int_{x^2}^{5}\sqrt{2t+1}\,dt$, then $f'(x)=$

a) $\sqrt{2x^2+1}$ b) $-\sqrt{2x^2+1}$ c) $2x\sqrt{2x^2+1}$ d) $-2x\sqrt{2x^2+1}$ e) NOTA

17. If $y=f(x^2)$ and $f'(x)=\sqrt{5x-1}$, then $\dfrac{dy}{dx}=$

a) $f'(x^2)$ b) $\sqrt{5x^2-1}$ c) $x\sqrt{5x^2-1}$ d) $2x\sqrt{5x^2-1}$ e) NOTA

18. The denominator of a fraction exceeds the square of its numerator by 16. The maximum value of the fraction is :

a) $\dfrac{3}{16}$ b) $\dfrac{5}{8}$ c) $\dfrac{-4}{33}$ d) $\dfrac{5}{24}$ e) NOTA

19. For what value of k will $x+\dfrac{k}{x}$ have a relative maximum at x = -2?

a) –4 b) –2 c) 2 d) 4 e) NOTA

20. $\lim_{n\to\infty}\dfrac{3}{n^2}(2+4+6+\ldots+2n)=$

a) 2 b) 3 c) 4 d) 6 e) NOTA

21. Use differentials to approximate the decrease in the volume of a cube when the side is decreased from 8 to 7.99.

a) 19.2 b) 15.36 c) 1.92 d) 0.01 e) NOTA

22. A particle moves in a straight line with velocity v (t) = t^2. How far does the particle move between times t = 1 and t = 2?

a) 1/3 b) 7/3 c) 3 d) 7 e) NOTA

23. Let f and g be differentiable functions such that: $f(1)=2, f'(1)=3, f'(2)=-4$ and $g(1)=2, g'(1)=-3, g'(2)=5$. If $h(x)=f(g(x))$, then $h'(1)=$

a) –9 b) –4 c) 0 d) 12 e) NOTA

24. What is the work (in ft.-lbs.) done by a force (in lbs.), propelling a particle along the x-axis from x = 1 to x = 4 (in ft.), if $F=\frac{1}{\sqrt{x}}$?

a) 1 b) 2 c) 4 d) 8 e) NOTA

25. The position of a particle moving along the x-axis at time t is given by $x(t)=e^{\cos(2t)}$. For which of the following values of t will $x'(t)=0$?

I. t = 0 II. $t=\frac{\pi}{2}$ III. $t=\pi$

a) I only b) II only c) I & III only d) I & II only e) NOTA

26. The equation of the line *normal* to $y=\sqrt{\frac{5-x^2}{5+x^2}}$ at $x=2$ is

a) $81x-60y=142$ b) $81x+60y=182$ c) $20x+27y=49$ d) $20x+27y=31$ e) NOTA

27. $\int_{\frac{2}{\pi}}^{\infty} \frac{\sin\left(\frac{1}{t}\right)}{t^2} dt =$

a) 1 b) 0 c) –1 d) 2 e) NOTA

28. A company manufactures x calculators weekly that can be sold for 75 - .01x dollars each, at a cost of $1850 + 28x - x^2 + 0.001x^3$ dollars for manufacturing x calculators. The number of calculators the company should manufacture weekly in order to maximize its weekly profit is?

a) 611 b) 652 c) 683 d) 749 e) NOTA

29. The rate that an object cools is directly proportional to the difference between its temperature (in Kelvin) at that time and the surrounding temperature (in Kelvin). If an object is initially at 35K, and the surrounding temperature remains constant at 10K, it takes 5 minutes for the object to cool to 25K. How long will it take for the object to cool to 20K (to the nearest 100^{th})?

a) 6.66 min b) 7.50 min c) 7.52 min d) 8.97 min e) NOTA

30. Two particles leave the origin at the same time and move along the y-axis with their respective positions determined by the functions $y_1 = \cos 2t$ and $y_2 = 4\sin t$ for $0 < t < 6$. For how many values of t do the particles have the same acceleration?

a) 0 b) 1 c) 2 d) 3 e) NOTA