February 1998 FAMAT Regional Mathematics Competition
Calculus Individual Test
If none of the given answers is correct, choose E. NOTA (None of the above)

1. Find the domain of the equation $y = \left(\sqrt{12 - x - x^2}\right)^{-1}$.
A. $(-\infty, -3) \cup (4, \infty)$ B. $(-\infty, -4) \cup (3, \infty)$ C. $(-3, 4)$ D. $(-4, 3)$ E. NOTA

2. Find the slope of a line tangent to $y = (\cos^3 x)(\sin x)$ at $x = \pi/3$.
A. –0.5 B. 0 C. 0.625 D. 1.125 E. NOTA

3. Find the maximum value of the function $y = x^3 + 3x^2 - x - 3$ on the interval [-4, 2].
A. –15 B. $\frac{-16\sqrt{3}}{9}$ C. 0 D. $\frac{16\sqrt{3}}{9}$ E. NOTA

4. Find all asymptotes for the graph $\frac{1}{y} = \frac{2x^2 - 3x + 1}{x^2 - 7x + 12}$.
A. $y = 2, x = 3, x = 4$ B. $y = 0, x = 3, x = 4$ C. $y = 0.5, x = 0.5, x = 1$
D. $y = 0.5, x = 0.5, x = 1, x = 3, x = 4$ E. NOTA

5. What is the value of dy/dx at $x = 2$ if $y = 2^x + \log_2 x$.
A. 8.5 B. $8 + \frac{1}{\ln 4}$ C. $\frac{9 \ln 2}{2}$ D. $4 \ln 2 + \frac{1}{\ln 4}$ E. NOTA

6. Let a be the number of the following functions that are odd. Let b equal the number of the following functions that are even. Find a-b.
$y = x$ $y = |x|$ $y = \lfloor x \rfloor$ $y = \sqrt{x}$ $y = |x|^{|x|}$ $y = x^3 + \sin x$ $y = \cos x$
A. –1 B. 0 C. 1 D. 2 E. NOTA

7. Find the area of the region bounded by the x-axis, the y-axis, and the graphs $y = x^2 - 2x + 3$ and $y = 5 - x$.
A. 10/3 B. 47/6 C. 55/6 D. 115/6 E. NOTA

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8. Use differentials to approximate $\sqrt[3]{1704}$ using the fact that $12^3 = 1728$.
A. 34/3 B. 23/2 C. 71/6 D. 215/18 E. NOTA

9. Find the limit as x approaches infinity of $\sqrt{x^2+6x+3} - \sqrt{x^2+1998}$.
A. –2004 B. –1995 C. 0 D. $+\infty$ E. NOTA

10. Find the equation of the line normal to the graph $y = x^3 - 6x^2 + 3x - 3$ at $x = 2$.
A. $y = 9x - 31$ B. $x = 9y + 119$ C. $9x + y = 5$
D. $x + 9y + 115 = 0$ E. NOTA

11. Find $f^{(1998)}(x)$ for the equation $f(x) = \frac{x^{1998}}{1998!} + 1998e^x + 1998 + \frac{1}{x}$.
A. $1 + 1998e^x + 1998x^{1998}$ B. $1 + 1998e^x + 1998!x^{-1998}$
C. $x + 1998e^x + 1998 + x^{-1998}$ D. $x + 1998!e^x + 1998!x^{-1998}$
E. NOTA

12. Evaluate the limit as x approaches infinity of $\left(2 + \frac{3-x}{x}\right)^{x/5}$.
A. $(2+e^3)^{1/5}$ B. $(1+e^3)^{1/5}$ C. $e^{1/15}$ D. $e^{3/5}$ E. NOTA

13. Donna is located 1000 feet directly north of Dave. Donna travels directly North at 100 feet per second. At the same time, Dave travels Southeast at 50 feet per second. How fast is the distance between Donna and Dave changing 5 seconds after they both start moving? (Rounded to the nearest foot per second)
A. 127 B. 138 C. 150 D. 277 E. NOTA

14. Determine the limit as n approaches 0 of $\frac{nx^2 + 3nx + n^2}{nx + 4n^3 + 3n}$ for $x > 0$.
A. 0 B. 0.75 C. 1 D. x E. NOTA

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15. Find the area of the region bounded by the graphs $y = x^2 - 7$ and $y = 25 - x^2$ within the first quadrant.

A. $\frac{14\sqrt{7}}{3}$ B. $\frac{14\sqrt{7}}{3} - 2$ C. $\frac{256 - 14\sqrt{7}}{3}$ D. $\frac{512}{3}$ E. NOTA

16. Find dy/dx if $y = (\sin x)(\cos x)(\tan x)$.

A. $(\sin x)(\cos x) + \tan x + (\sin x)^2(\tan x)$ B. $(\sin x)^2(\tan x) + (\sin x)(\cos x)$
C. $(\sin x)(\cos x)$ D. 0 E. NOTA

17. A projectile is launched from the top of a cliff with an initial upward velocity of 80 feet per second. With what downward velocity (rounded to the nearest integer) will the projectile impact the ground 500 feet below the peak of the cliff. Use $32 ft/sec^2$ for the acceleration due to gravity.

A. 196 B. 265 C. 276 D. 345 E. NOTA

18. The radius of a sphere is increasing at a constant rate of 1 inch per second. What is the ratio of the rate at which the volume is increasing to the rate at which the surface areas is increasing when the length of the diameter of the sphere is 8 inches.

A. 2 B. 4 C. 4π-2 D. 4π E. NOTA

19. At the end of 1980, Macrohard employed 8000 individuals. The rate at which the employee head count grows is directly proportional to the current head count. At the end of 1997, there were 25,780 employees at Macrohard. If the employee growth rate maintains the same growth rate proportion, how many employees will work for Macrohard at the end of the 20^{th} century? (round your answer to the nearest integer)

A. 27872 B. 28918 C. 29585 D. 31693 E. NOTA

20. Evaluate the derivative of $\sin^2 x$ with respect to $\sin x$ at $x = \pi/2$.

A. 0 B. 1 C. 2 D. undefined E. NOTA

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21. Find the sum of the y-coordinates of all relative extrema for the graph $y = 2x^6 - 21x^4 + 8x^3 + 72x^2 - 96x + 1998$ between x = -10 and x = 10.

A. 1950 B. 6119 C. 3600346 D. 3604515 E. NOTA

22. Evaluate dy/dx at x = 0.5 for the equation $y = \arcsin x + x$.

A. $\frac{3+\sqrt{3}}{3}$ B. $\frac{3+2\sqrt{3}}{3}$ C. $2\sqrt{3}+1$ D. $\frac{\pi+3}{3}$ E. NOTA

23. $f(x) = 2x^3 + 6x - 27$ and $g(x) = f^{-1}(x)$. Find $g'(1)$.

A. 0.5 B. 2 C. 12 D. 30 E. NOTA

24. $\frac{dy}{dx} = \frac{\frac{1}{y}(2x+3)}{3y+2}$. y = 1 when x = 0. Determine the sum of all possible values for x when y = 0.

A. –3 B. 2 C. 3 D. undefined E. NOTA

25. If $f(x) = xe^x$, then what is $f^{(1998)}(0)$?

A. 0 B. 1 C. 1997 D. 1998 E. NOTA

26. Evaluate dy/dx at y = 1 for the equation $3 - y^2 = \frac{xy^3 + y^2 + 2xy + 8}{7x - y^2}$.

A. 0 B. $\frac{11}{23}$ C. 0.5 D. undefined E. NOTA

27. Find the volume of the solid formed by revolving the graph $y = x^2$ about the y-axis from x = 0 to x = 2.

A. 6.4π B. 8π C. 128π D. 204.8π E. NOTA

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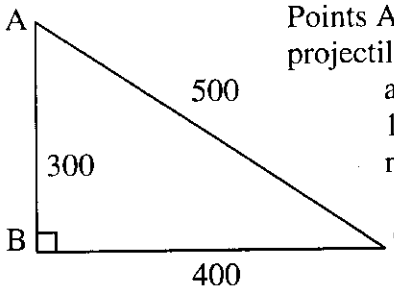
28. Rectangle WHAT has vertices (1, 3), (5, 1), (4, 9), and (8, 7). WHAT is rotated about one of the coordinate axes to form a solid. What is the positive difference between the volumes of the two possible solids formed in this fashion?

A. 7π B. 15π C. 30π D. $7\pi\sqrt{3}$ E. NOTA

29. Which of the following is equivalent to the limit as n approaches infinity of $\sum_{x=1}^{n}\left(\frac{(x+n)^2}{n^3}\right)$?

A. $\int_1^{\infty}\left(\frac{1}{x}+1\right)^2 dx$ B. $\int_0^1 \frac{(x+1)^2 dx}{x}$ C. $\int_1^{\infty} \frac{(x+1)^2 dx}{x}$

D. $\int_0^1 (x+1)^2 dx$ E. NOTA

30. 

Points A, B, and C are on a level suface as shown. A projectile is fired from B towards A. Its initial horizontal and vertical velocities are 30 feet per second and 160 feet per second, respectively. Ignore air resistance and use 32 ft/sec^2 for the acceleration due to gravity. Rachel runs from C to A at 50 feet per second beginning at the same instant that the projectile is fired. How fast is the distance between Rachel and the projectile decreasing 5 seconds after they begin moving?

A. $\frac{31\sqrt{5}}{5}$ B. $8\sqrt{5}$ C. $\frac{39\sqrt{5}}{5}$ D. $56\sqrt{5}$ E. NOTA