

## PreCalculus Team Questions Regional, 2001

1. Evaluate  $(1+i)^{16}$ .
2. Find the arithmetic average of the roots of the equation  $x^3 - 7x^2 + 10x = 0$ .
3. Given the equation  $\frac{5}{8}x - \frac{3}{7}y + \frac{7}{11} = 0$ . If A is the slope of the equation and B is the y-intercept. Evaluate  $\frac{A}{B}$ .
4. Evaluate the following limit  $\lim_{x \rightarrow 0} \csc(2x) \tan(3x)$ .
5. John takes 4 hours to paint a house. Phil takes 6 hours to paint the same house. They work together to complete the task faster. If Phil gets paid \$12/hr and the entire job costs \$72, what is John's hourly rate? Express your answer in dollars and cents, to the nearest cent.
6. Find the sum of all the sides of all Pythagorean triples, where the smallest side is less than 10. (A Pythagorean triple is a right triangle with all integral sides.)
7. Adam asks David for change of \$1.00. David reaches deep inside his pocket and pulls out all his change. He says "Gee, I can't give you exact change, in fact I have the most money I can have without being able to give you change of a dollar." How much money was David holding. (He had only American money which had some or all of the following, quarters, dimes, nickels, and cents.)
8. Given the following, evaluate the determinant of  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Leave your answer in fractional form.

$$a = 1 + \frac{2}{3 + \frac{4}{5}}$$

$$b = \text{sum of the roots of } y = x^2 - 19x + 84$$

$$c = \text{sum of the infinite geometric series whose first term is } \frac{3}{11} \text{ and whose ratio is } \frac{2}{19}$$

$$d = \text{sum of the first 7 prime numbers}$$

9. On Monday there is a snail 20 ft below the top of a well. Each day he crawls up 7 inches, but each night he slides back 2 inches. Every Sunday the snail rests, and slides back a total of 3 inches (this is the amount for the day and the night total movement on Sunday). On what day of the week will the snail escape the well?

10. Solve for x, y, and z, and then find the determinant.

$$\begin{aligned} x + 2y + 5z &= 33 \\ 2x - y + 3z &= 16 \\ -x + 3y - 2z &= -3 \end{aligned}$$

$$\begin{bmatrix} 1 & x \\ y & z \end{bmatrix}$$

11. An 8-sided and a 12-sided number generator are tossed. The numbers from 1 to 8 appear on one side of the 8-sided number generator. The numbers from 1 to 12 appear on one side of the 12-sided number generator. Both number generators are fair, so each number is just as likely to be tossed as any other. The result is the sum of the numbers tossed on both. What is the probability that the result will be a prime number?

12. Given the information below, find where the following lines intersect.

$$Ax + By + 1 = 0 \quad \text{and} \quad Cx + Dy + 2 = 0$$

- A = number of primes less than 32
- B = product of the first 3 primes
- C = number of vertices on a cube
- D = sum of the primes less than 10

Write your coordinates in fractional form.

13. At Mu Alpha Theta banquet dinner starts at precisely 7:02 on the dot. After 17 minutes and 36 seconds 40% of the baked ziti has been eaten. If the baked ziti is eaten at a constant rate what time will it be to the nearest second when only 10% of the baked ziti remains?

14. Find the determinant of  $\begin{bmatrix} 3 & 4 & 17 \\ 0 & 2 & 7 \\ 0 & 3 & 11 \end{bmatrix}$ .

15. A car goes around a one mile track at 45 m.p.h. How fast must the car travel on the second lap so the average speed for both laps will be 60 m.p.h.?