

# Precalculus Team Questions

## FAMAT Regional Competition

### Answers

1.  $\frac{\sqrt{35}}{6}$

2.  $\frac{7\pi}{4}$

3. 12

4. 24

5. 1.6

6. 2.6

7. 4.8

8. 21

9. 23

10.  $406 + 2\sqrt{3}$

11. 12.5% (percent sign not necessary)

12. 948.7

13. 1.9

14. 121

15. 12.4

Solutions: Precalculus Team Questions  
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1.  $\frac{1}{2}(\sin \theta \cos \theta) \tan \theta = \frac{1}{72}$ ;  $\frac{1}{36} = \sin^2 \theta$ ; since  $\theta$  is in quadrant I, we have  $\sin \theta = \frac{1}{6}$   
and  $\cos \theta = \frac{\sqrt{35}}{6}$ . Answer is  $\frac{\sqrt{35}}{6}$ .

2. We use the third part of the function:  $\cos x = \frac{\sqrt{2}}{2}$  for  $\frac{7\pi}{4}$ . The answer is  $\frac{7\pi}{4}$ .

3.  $\frac{1}{2}bh = \frac{1}{2}k^2 \sin \theta \cos \theta = 9$  and  $\sin \theta \cos \theta = \frac{18}{k^2}$  so  $2 \sin \theta \cos \theta = \frac{36}{k^2}$

Since the left part of the equation is the identity for  $\sin(2\theta)$  we have  $\frac{1}{4} = \frac{36}{k^2}$  and since  $k$  is positive we have  $k = 12$ . The answer is 12.

4. I have won 96 games.  $\frac{\text{wins}}{\text{played}} = \frac{96+x}{120+2x} = \frac{3}{4}$  and this solves to  $x=12$  games; the answer is 24.

5. Arc AB has measure  $2\theta$  and length  $\frac{2\theta}{2\pi}(2\pi(10 \cos \theta))$ . Setting this equal to  $\frac{\theta}{18}$  we get that  $\cos \theta = \frac{1}{360}$  and so we approximate  $\theta$  to 1.6. The answer is 1.6

6. Using the law of cosines,  $16 + 36 - 2(4)(6)\cos A = 81$  and  $16 + 25 - 2(4)(5)\cos B = 64$ .  
So we get  $A \approx 127.6888997$ ;  $B \approx 125.0996$  and their net difference is approx. 2.6

7.  $m\angle C = \text{Arctan} \frac{6}{8}$  and to find BD we use  $\sin(C) = \frac{BD}{8}$ , obtaining  $BD = 4.8$

8.  $A=9, B=8, C=1, D=3$ . Sum = 21

9. Sum of the roots is 6 and the product is  $9 - i^2 = 10$ . So the quadratic function  $f(x) = k(x^2 - 6x + 10)$ . Since  $f(2)=4$ , we have  $4 = k(4 - 12 + 10)$  and  $k=2$ .

So  $f(x) = 2x^2 - 12x + 20$  and  $A=f(0)=20$ ;  $B = \frac{-b}{2a} = \frac{12}{4} = 3$ .  $A+B = 23$

10.  $A = 120\left(\frac{\pi}{180}\right) = \frac{2\pi}{3}$ ;  $B = \pi(1 + \sqrt{3})^2 = (4 + 2\sqrt{3})\pi$ ;  $C = 10^{\log 400} = 400$   
 $\frac{3A+B}{\pi} + C = 2 + (4 + 2\sqrt{3}) + 400 = 406 + 2\sqrt{3}$ . The answer is  $406 + 2\sqrt{3}$ .

11. Coffee =  $\pi r^2 h = \pi(9)(6)$  and cream is  $\pi(9)(2)$  so cream percent is  $\frac{\pi(9)(2)}{\pi(9)(8)} = 25\%$

When liquid is drunk, the cream is then  $\pi(9)(4)(0.25)$  and after coffee is added the percent is  $\frac{\pi(9)(4)(0.25)}{\pi(9)(8)}$  which is 0.125 or 12.5%

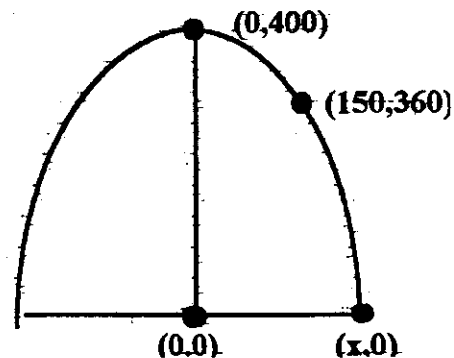
12.  $y - 400 = a(x^2)$  and so

$360 - 400 = a(150^2)$

and  $a = -\frac{40}{150^2}$ . Substituting, we get

$-400 = \frac{-40}{150^2}x^2$  which solves to 474.341649 for

the positive  $x$  value of the foot. The distance between the feet is 948.7



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13. The rate of point P is  $(0.2)(20\pi) = 4\pi$  and that of point Q is  $(0.3)(20\pi) = 6\pi$ .  
The circumference of the circle is  $20\pi$  so the distance to be traveled is  $19\pi$ , and  
 $19\pi = (4\pi)d + (6\pi)d$  by use of  $tr=d$ . Solving gives that the points will meet in 1.9  
minutes. The answer is 1.9
14. Since  $\sin \alpha = \frac{3}{5}$  then  $\cos \alpha = \frac{4}{5}$ ; since  $\cos \beta = \frac{8}{17}$  then  $\sin \beta = -\frac{15}{17}$  in the quadrant stated.  
 $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta = \frac{3}{5} \cdot \frac{8}{17} + \frac{4}{5} \cdot \frac{-15}{17} = \frac{-36}{85}$  and  $|k| + |m| = 121$
15. The rectangular equivalents are  $(5, 5\sqrt{3})$ ,  $(0, 20)$  so the distance is  
 $\sqrt{(5^2 + (20 - 5\sqrt{3})^2)} \approx 12.4$