

Pre-Calculus Question 1

February 8, 1992

Find the sum of all solutions for x , where $0 \leq x \leq \pi$.

$$2 \tan^2 3x + 3 = 3 \sec 3x$$

Pre-Calculus Question 2

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A = the length in inches of the diagonal of a right rectangular solid with dimensions 3 inches by 4 inches by 1 foot

B = the area of the triangle formed by the lines whose equations are $x + y = 16$, $x = 3y$, and $y = 0$

C = the total surface area (in square inches) of a cube with a 6" edge, if the cube has a right circular cylindrical hole with a 2 inch diameter cut through it from one face to the opposite face

Find $A + B + C$.

Pre-Calculus Question 3

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Find the positive value of k such that one root of $x^3 - 63x + k = 0$ is half of another.

Pre-Calculus Question 4

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(Note: $i^2 = -1$)

A = the reciprocal of $(1 + i)(1 + 2i)$

$$B = i^3 + i^4 + i^5 + \dots + i^{1992}$$

C = the sum of the real values of x and y for which the equation

$$(3x - 2y - 13) - i(4x + 5y) = \frac{2}{i}$$

Find $10A + B + C$ in the form $a + bi$ where a and b are real numbers.

Pre-Calculus Question 5

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Find the length of an external tangent drawn to the circle $x^2 + y^2 - 4x + 8y - 5 = 0$ from the point $(7, 8)$.

Pre-Calculus Question 6

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$A =$ the number of terms in the expansion of $[(a + 2b)^3(a - 2b)^3]^4$

$B =$ the sum of the coefficients in the expansion of $(4x - 2y)^6$

$C =$ the coefficient of the sixth term in the expansion of $\left(3x + \frac{1}{2}y\right)^8$

Find ABC .

Pre-Calculus Question 7

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Evaluate in simplest form:

$$\sin\left[\frac{1}{2}\text{Arc tan}\frac{8}{15}\right]$$

Pre-Calculus Question 8

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$$A = \log_3(2\sqrt{75} - \sqrt{147})$$

$$B = \text{the value of } x \text{ such that } 4^{\log_2 3} = 8x + 5$$

$$C = \text{the value of } y \text{ such that } \log_5(y-1) + \log_5(y-2) = \log_5(y+7)$$

$$D = \text{the value of } z \text{ such that } \log_2(\log_3(\log_4 z)) = 0$$

Find $ABCD$.

Pre-Calculus Question 9

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$$f(x) = \frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x}$$

$$g(x) = \frac{\sec x + \tan x}{\cos x - \tan x - \sec x}$$

$$h(x) = f(x) \cdot g(x)$$

Find $h\left(\frac{\pi}{8}\right)$ in simplest form.

Pre-Calculus Question 10

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A = the number of distinct permutations of the letters in the word CALCULUS

B = the number of ways 6 keys can be arranged on a key ring

C = the number of ways 8 people can be divided into two groups of equal size

D = the number of distinct permutations of the letters in the word GEOMETRY that both begin and end with an "E".

Find $\frac{AB}{CD}$.

Pre-Calculus Question 11

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Find the sum: $\sum_{x=-15}^{15} x^3 + x^2 + x + 1$

Pre-Calculus Question 12

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Find the value of $\frac{3wx - yz}{2yz - wx}$ if $\frac{w}{y} = \frac{2}{3}$ and $\frac{x}{z} = \frac{1}{4}$.

Pre-Calculus Question 13

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A certain circle has a chord of length 12 which is tangent to a smaller, concentric circle.
Find the area between the circles.

Pre-Calculus Question 14

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Solve for all real values of x : $\sqrt{2x} - \sqrt{x-3} = \frac{2}{\sqrt{x-3}}$

Pre-Calculus Question 15

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Given that line #1 is determined by the points (5, 6) and (7, 10) and that line #2 is determined by the points (-1, 8) and (3, -5), find the sum of the coordinates of the point of intersection of these two lines.