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FAMAT Regional Math Competition 2-9-91
Pre-Calculus Team Round Question 1

1. Determine the value of c so that the given circle has a radius of 2: $x^2 - 6x + y^2 + 8y + c = 0$.
2. Determine the value of d so that the line through $(-2,3)$ and $(5,d)$ is perpendicular to $2x - 3y = 5$.

Find $\frac{c}{d^2}$.

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Solve for x :
$$\frac{\tan(x + \frac{2\pi}{9}) + \tan(\frac{\pi}{9} - x)}{1 - \tan(x + \frac{2\pi}{9})\tan(\frac{\pi}{9} - x)} = -2\cos x$$

Note: $0 \leq x \leq 2\pi$

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FAMAT Regional Math Competition 2-9-91
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Let $A = \sum_{k=1}^{21} (-1)^k \cos(\pi)$

Let B be the number of layers needed to pile 150 toothpicks in layers so that each layer has 1 less toothpick than the layer below and the first layer has 3 toothpicks.

Let C be the sum of the infinite geometric series

$$3\sqrt{5} + \frac{3\sqrt{10}}{2} + \frac{3\sqrt{5}}{2} + \dots$$

Find $A + B + C$.

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FAMAT Regional Math Competition 2-9-91
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$$\text{Let } A = \log_{\sin \Theta} \left[\frac{\sin \Theta}{\sin \Theta - (\sin \Theta)(\cos^2 \Theta)} \right].$$

Let B be the value of $\frac{e^{3 \ln x} + e^{3 \ln 2}}{e^{\ln(x+2)}}$ when $x = 4$.

$$\text{Let } C = \frac{\log_4 8}{2} + \frac{2 \log_2 \left(\frac{1}{4}\right)}{3} - \log_8 \sqrt{2}.$$

Find $\frac{AB}{C}$.

5

FAMAT Regional Math Competition 2-9-91
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In a survey of 100 students it was determined that 43 listened to WXKL, 48 to WTRS, 38 to WTSJ;
20 listened to WXKL and WTSJ
31 listened to WTRS and WTSJ
28 listened to WXKL and WTRS
15 listened to all 3.

If one of these surveyed students is selected at random, find the probability that the student does not listen to WTRS.

6

FAMAT Regional Math Competition 2-9-91
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Find the intersection points of $\log_4 y = x^2 - 3$ and $y = 8^{(x+1)}$.

Do not write your answer in exponential form!

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For what value k does this system of equations have no solution?
 $x - 3y - 2z = 0$; $x + ky = 1$; $2x + ky + z = 1$

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Let A be the solution to the following equation in mod 11:

$$9x + 6 = 2x + 10.$$

Let B and C be the solutions to the following equation in mod 7:

$$x^2 + 2x + 6 = 0.$$

Find $\frac{BC}{A}$.

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Solve for D : $4D - 3AC = BA^{-1} - 2CB$

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$$

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Find the range of the following function:

$$f(x) = \left| |2\sin(x)| - 3 \right|.$$

11

FAMAT Regional Math Competition 2-9-91
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Find all solutions for x over the complex numbers:

$$x(x^3 + 27) - 15(x^2 + 6x + 9)(x^2 - 3x + 9) = 0.$$

12

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Find the point of intersection of the conic $(x - 7)^2 + y^2 = 13$ with the line of positive slope which is an asymptote of

$$9(x-2)^2 - 4(y+1)^2 = 36$$

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Given vectors $v = \langle 1, 2, -1 \rangle$ and $w = \langle 2, 0, -3 \rangle$. Find the magnitude of their cross product ($v \times w$). Divide your answer by the dot product ($v \cdot w$).

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FAMAT Regional Math Competition 2-9-91
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Let A be the domain of $y = \frac{\sqrt{x-2}}{x-3}$.

Let B be the solution set for $|x+3| < 2x-1$.

Let C be the solution set for $\log_2(x^2-4) \geq 3$.

Find $(A \cup B) \cup C$.

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FAMAT Regional Math Competition 2-9-91
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Three circles each with radius 6 are tangent to each other. Find the area of the region enclosed between them.