Lehigh '96

Pre-Calculus Individual - Solutions

1) $y = x^2 - 6x + 5$

$y = (x^2 - 6x + 9) + 5 - 9$

$y = (x+3)^2 - 4$

$\boxed{\text{Range} : [-4, \infty)}$

Vertex $(-3, -4)$

2) amplitude: 2

$P = \frac{2\pi}{b} \Rightarrow \pi = \frac{2\pi}{b}$

$b = 2$

$y = a \sin bx$

$\boxed{y = 2 \sin 2x}$

3) $2 \log_5 x + \log_5 4 = \log_5 36$

$\log_5 4x^2 = \log_5 36$

$4x^2 = 36$

$x^2 = 9$

$x = \pm 3$ so $\boxed{x = 3}$

4) $\sim(p \wedge q) = \sim p \vee \sim q$

$\Rightarrow$ Today is not cold or not snowy.

(includes 0)

E

5) $\sec^2 x = (1+\sqrt{3}) - (1-\sqrt{3})\tan x$

$\tan^2 x + \cancel{1} = \cancel{1} + \sqrt{3} - (1-\sqrt{3})\tan x$

$\tan^2 x + (1-\sqrt{3})\tan x - \sqrt{3} = 0$

$(\tan x + 1)(\tan x - \sqrt{3}) = 0$

$\tan x = -1 \qquad\qquad \tan x = \sqrt{3}$

$x = \frac{3\pi}{4}, \frac{7\pi}{4} \qquad\qquad x = \frac{\pi}{3}, \frac{4\pi}{3}$

$$\boxed{x = \frac{\pi}{3}, \frac{3\pi}{4}, \frac{4\pi}{3}, \frac{7\pi}{4}}$$

6) $(x-3)^2(x-7) \le 0$ iff $x - 7 \le 0$

$$\boxed{x \le 7}$$

7) $2\sin^2 x + \sin x - 1 < 0$

$(2\sin x - 1)(\sin x + 1) < 0$

$\sin x < \frac{1}{2} \qquad\qquad \sin x < -1$

$x < \frac{\pi}{6}, \frac{5\pi}{6} \qquad\qquad x < \frac{3\pi}{2}$

$$\boxed{0 < x < \frac{\pi}{6} \quad \cancel{\text{OR}} \quad \frac{5\pi}{6} < x < \frac{3\pi}{2} \quad \text{OR} \quad \frac{3\pi}{2} < x < 2\pi}$$

8) The graph crosses the line $y = 2$ in 3 places; therefore 3 solutions.

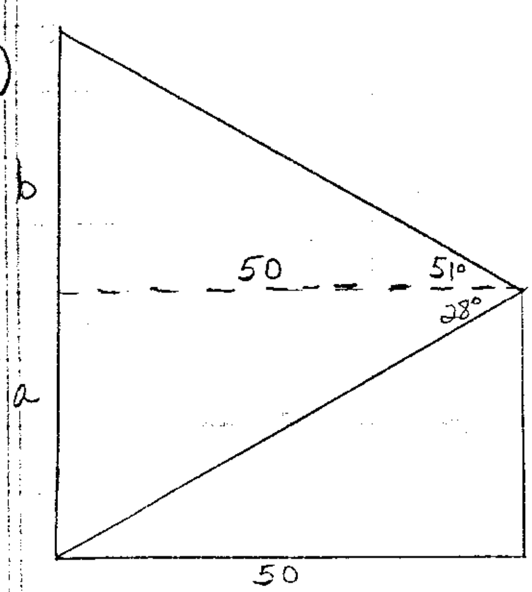
9) $\dfrac{x+5}{x-1} - \dfrac{x^2+2}{x^2+2x-3}$

$x^2+2x-3 = (x-1)(x+3)$

$\dfrac{(x+5)(x+3) - (x^2+2)}{(x-1)(x+3)}$

$\dfrac{\cancel{x^2}+8x+15-\cancel{x^2}-2}{x^2+2x-3}$

$\boxed{\dfrac{8x+13}{x^2+2x-3}}$

10) 

$\tan 28° = \dfrac{a}{50}$

$a = 50 \tan 28°$

$\tan 51° = \dfrac{b}{50}$

$b = 50 \tan 51°$

$h = a + b$

$= 50 \tan 28° + 50 \tan 51°$

$\boxed{\approx 88 \text{ feet}}$

11) $2x^3 + 7x^2 - 77x - 40 = (x-5)(2x^2 + 17x + 8)$
$= (x-5)\boxed{(2x+1)}(x+8)$

5	2	7	-77	-40
		10	85	40
	2	17	8	0

12) $f(x) = \frac{x-5}{\sqrt{x-2}}$

$x - 2 > 0$
$x > 2$
$\boxed{\text{Domain: } (2, \infty)}$

13) $d = KF$
$-1.9 \text{ in.} = K(25 \text{ lb.})$
$K = -0.076 \frac{\text{in}}{\text{lb}}$

$d = -0.076\ F$
If $d \geq -3$, then $-3 \leq -0.076\ F$
$F \leq \frac{3}{0.076}$
$\boxed{\approx 39.5 \text{ lb.}}$

14) $2^x = 3^{2x+1}$

$\ln 2^x = \ln 3^{2x+1}$

$x \ln 2 = (2x+1) \ln 3$

$x \ln 2 = 2x \ln 3 + \ln 3$

$x \ln 2 - 2x \ln 3 = \ln 3$

$x(\ln 2 - 2\ln 3) = \ln 3$

$x = \dfrac{\ln 3}{\ln 2 - 2\ln 3}$

$\boxed{x \approx -0.7304}$

15) $2x + 3y + 5 = 0$

$3y = -2x - 5$

$y = -\frac{2}{3}x - \frac{5}{3}$

$m = -\frac{2}{3}$

$m\perp = \frac{3}{2}$

$y - 3 = \frac{3}{2}(x - 1)$

$2y - 6 = 3x - 3$

$\boxed{3x - 2y + 3 = 0}$

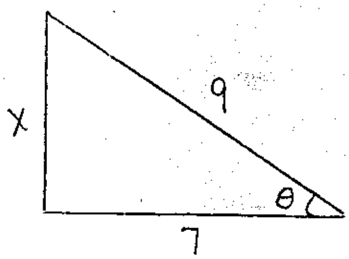
16) $A = Pe^{rt}$

effective yild is $e^{rt} \Rightarrow e^{.09} \approx 1.094$

$A = P(1+r)^t$

$= P(1.094)$ $\quad \therefore$ effective yild is $\boxed{9.4\%}$

17) If $\cos\theta = -\frac{7}{9}$ and $\tan\theta < 0$, angle is in quadrant II, so $\sin\theta > 0$.



$$x^2 + 7^2 = 9^2$$
$$x^2 = 81 - 49$$
$$x^2 = 32$$
$$x = 4\sqrt{2}$$

$\sin\theta = \frac{4\sqrt{2}}{9}$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{4\sqrt{2}}{9}\right)\left(-\frac{7}{9}\right)$$
$$\boxed{= -\frac{56\sqrt{2}}{81}}$$

18) $2\sin x\cos x + \cos x = 0$

$\cos x\,(2\sin x + 1) = 0$

$\cos x = 0$ $\quad$ $2\sin x = -1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$$\boxed{x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}}$$

19) $f(x) = \dfrac{-x^2-7x+27}{x(x^2+9)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+9}$

$A(x^2+9) + (Bx+C)(x) = -x^2-7x+27$

$Ax^2+9A+Bx^2+Cx = -x^2-7x+27$

$(A+B)x^2+Cx+9A = -x^2-7x+27$

$9A = 27$ $\quad Cx = -7x \quad A+B=-1$

$A = 3$ $\quad C = -7 \quad 3+B=-1$

$B = -4$

$\dfrac{3}{x} + \dfrac{-4x-7}{x^2+9}$ OR $\boxed{\dfrac{3}{x} - \dfrac{4x+7}{x^2+9}}$

20) $y = a(x-h)^2 + k$ $\quad \boxed{y = -\frac{1}{4}(x+1)^2+17}$

$1 = a(7+1)^2 + 17$

$1 = 64a + 17$

$-16 = 64a$

$-\frac{1}{4} = a$

21) $\ln x + \ln(x-2) = 1$

$\ln x(x-2) = 1$

$x^2-2x = e$

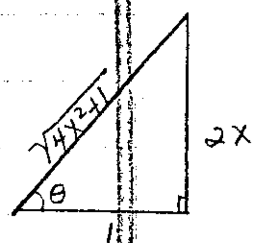
$x^2-2x-e=0$

$x \approx 2.93, -0.93$

x cannot be -0.93, $\therefore$ $\boxed{x \approx 2.93}$

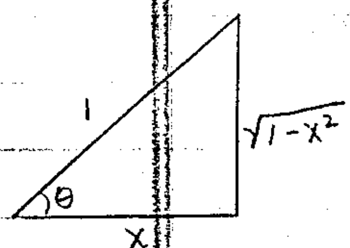
22) $\log_{20} 125 = \dfrac{\log 125}{\log 20} \approx \boxed{1.612}$

23) $\sin(\arctan 2x - \arccos x) = \sin(\arctan 2x)\cos(\arccos x) - \sin(\arccos x)\cos(\arctan 2x)$



$$= \left(\frac{2x}{\sqrt{1+4x^2}}\right)(x) - \left(\sqrt{1-x^2}\right)\left(\frac{1}{\sqrt{1+4x^2}}\right)$$

$$= \frac{2x^2}{\sqrt{1+4x^2}} - \frac{\sqrt{1-x^2}}{\sqrt{1+4x^2}}$$



$$\boxed{= \frac{2x^2 - \sqrt{1-x^2}}{\sqrt{1+4x^2}}}$$

24) $(\cos x + 1)^2 = (\sin x)^2$

$\cos^2 x + 2\cos x + 1 = \sin^2 x$

$\cos^2 x + 2\cos x + \cancel{1} = \cancel{1} - \cos^2 x$

$2\cos^2 x + 2\cos x = 0$

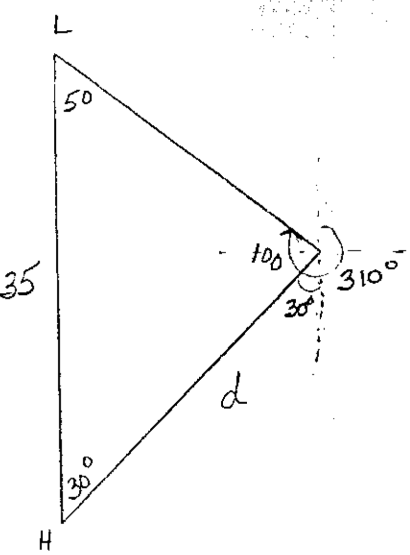
$2\cos x(\cos x + 1) = 0$

$\cos x = 0 \qquad \cos x = -1$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \pi$

But $\frac{3\pi}{2}$ is an extraneous solution.

$$\boxed{x = \frac{\pi}{2}, \pi}$$

25) 

$$\frac{\sin 100^\circ}{35} = \frac{\sin 50^\circ}{d}$$

$$d = \frac{35 \sin 50^\circ}{\sin 100^\circ}$$

$$d = 27.2$$

$$\text{speed} = \frac{27.2}{2} = \boxed{13.6}$$

26) $y = Ae^{Kt}$

$$\frac{A}{2} = Ae^{10K}$$

$$.5 = e^{10K}$$

$$\ln .5 = \ln e^{10K}$$

$$\ln .5 = 10K$$

$$\frac{\ln .5}{10} = K$$

$$\boxed{K = -0.069}$$

27) $16x^2 + 16y^2 + 16x + 40y - 7 = 0$

$$x^2 + y^2 + x + \frac{5}{2}y - \frac{7}{16} = 0$$

$$\left(x^2 + x + \frac{1}{4}\right) + \left(y^2 + \frac{5}{2}y + \frac{25}{16}\right) = \frac{7}{16} + \frac{1}{4} + \frac{25}{16}$$

$$\left(x + \frac{1}{2}\right)^2 + \left(y + \frac{5}{4}\right)^2 = \frac{9}{4}$$

$$r^2 = \frac{9}{4}$$

$$A = \pi r^2$$

$$\boxed{A = \frac{9\pi}{4}}$$

28) $f(x) = \frac{x+2}{x}$

$x = \frac{y+2}{y}$

$xy = y + 2$

$xy - y = 2$

$y(x-1) = 2$

$\boxed{y = \frac{2}{x-1} = f^{-1}(x)}$

29) $y = \frac{3x^2+2x-1}{x-1}$

$\boxed{y = 3x+5}$

$$\begin{array}{r l} & 3x + 5 \\ x-1 & \overline{)\,3x^2 + 2x - 1} \\ & \underline{3x^2 - 3x} \\ & \quad\quad 5x - 1 \\ & \quad\quad \underline{5x - 5} \\ & \quad\quad\quad 4 \end{array}$$

30) $3^{5x+1} = 5$

$\ln 3^{5x+1} = \ln 5$

$(5x+1)\ln 3 = \ln 5$

$5x \ln 3 + \ln 3 = \ln 5$

$5x \ln 3 = \ln 5 - \ln 3$

$5x = \frac{\ln 5 - \ln 3}{\ln 3}$

$x = \frac{\frac{\ln 5}{\ln 3} - 1}{5}$

$\boxed{x \approx 0.093}$