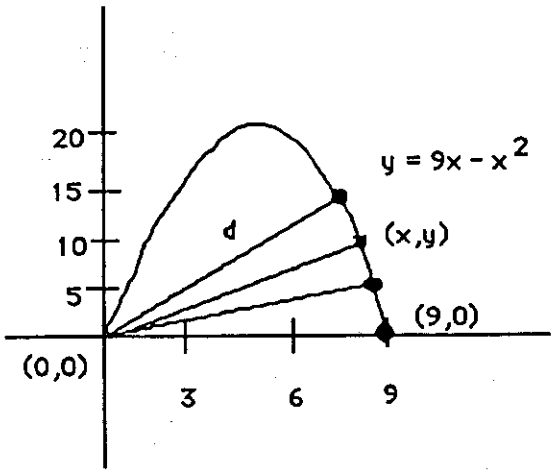
PRE-CALCULUS
REGIONAL COMPETITION

INDIVIDUAL TEST
FEBRUARY 8, 1994

ANSWER "E" IS "NOTA" NONE OF THE ABOVE

1: A projectile moves in a parabolic path given by $y = 9x - x^2$, as shown in the diagram. For any point (x,y) on the path, d, the distance from (x,y) to the launch point (0,0) is a function of x. Find the domain of d.



A: all real numbers B: $\{x: x \leq 9\}$ C: $(0, \infty)$
D: $[0,9]$ E: NOTA

2: Given $f(x) = \frac{1}{x}$, $x \in R$, $x \neq 0$. Find $f(f(2)) \cdot f(f(-3))$.

A: - 6 B: 6 C: 1/6 D: - 1/6 E: NOTA

3: f(x) = tan 2x, this function is:

i. symmetric with respect to the x-axis
ii. periodic
iii. monotonic
iv. bounded
v. continuous

A: i, ii and iii only B: i, ii and v only C: ii and iii only

D: ii, iii, and v only E: NOTA

4: Given the parametric equations $x = 3 \cos \varnothing$ and $y = 4 \sin \varnothing$, $0 \leq \varnothing \leq 2\pi$, the graph of this equation is a:

A: line B: ellipse C: circle
D: hyperbola E: NOTA

5: The equation of the lowest degree with real coefficients, one of whose roots is $\frac{2}{3} + \frac{i\sqrt{5}}{3}$ is:

A: $3x^2 - 4x + 1 = 0$ B: $3x^2 + 4x + 1 = 0$ C: $3x^2 - 4x + 3 = 0$
D: $3x^2 - 4x - 1 = 0$ E: NOTA

6: If $x^3 - kx + 8$ is exactly divisible by $x - 4$, the value of k is:

A: 2 B: 66 C: 6 D: -6 E: NOTA

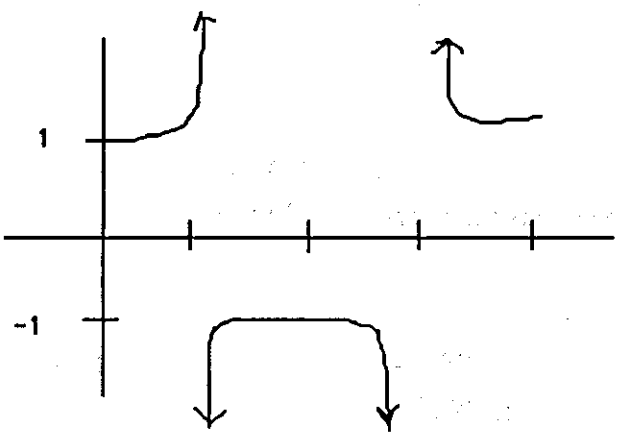
7: If y = arc cos x, the Domain and Range are

A: D = [-1,1] R = [-1,1]
B: D = $(-\infty, \infty)$ R = [-1 , 1]
C: D = $(-\infty, \infty)$ R = $(-\infty, \infty)$
D: D = [-1 , 1] R = $(-\infty, \infty)$
E: NOTA

8: Which of the following does not equal cos 2x?

A: $\cos^2 x + \sin^2 x$
B: $\cos^2 x - \sin^2 x$
C: $1 - 2\sin^2 x$
D: $2\cos^2 x - 1$
E: NOTA

9:



With the domain shown,
This is the graph of

A: y = arcsin x
B: y = sec x
C: y = csc x
D: y = arc cos x
E: NOTA

10: $\frac{\text{Cos x + 1}}{\text{Sin}^3 \text{ x}} =$

A: sin x + cos x

B: $\frac{1 - 3 \sin x}{1 - \sin x}$

C: $\frac{\sec x - \sin x}{\tan x - 1}$

D: $\frac{\csc x}{1 - \cos x}$

E: NOTA

11: The exact value of tan 165° is:

A: $-2 - \sqrt{3}$

B: $\frac{\sqrt{3}}{6}$

C: $\frac{\sqrt{3}}{2}$

D: $-2 + \sqrt{3}$

E: NOTA

11. A car travels at a rate of 30 miles an hour up a grade which makes an angle of 10° with the horizontal. How long does it take the car to rise 200 feet?

A: 1151.75 ft/sec

B: 38.39 sec

C: 44 sec

D: 26.2 sec

E: NOTA

13: Solving the following system of equations for y with Cramer's Rule involves using a numerator equal to which determinant?

$3x - 2y + 5z = 0$
$6x + y = 4$
$2x + 3y - 8z = 4$

A: $\begin{vmatrix} 3 & -2 & 5 \\ 6 & 1 & 0 \\ 2 & 3 & -8 \end{vmatrix}$ B: $\begin{vmatrix} 3 & -2 & 0 \\ 6 & 1 & 4 \\ 2 & 3 & 4 \end{vmatrix}$ C: $\begin{vmatrix} 3 & 0 & 5 \\ 6 & 4 & 0 \\ 2 & 4 & -8 \end{vmatrix}$

D: $\begin{vmatrix} 0 & -2 & 5 \\ 4 & 1 & 0 \\ 4 & 3 & -8 \end{vmatrix}$ E: NOTA

14: $\log_4 \{(3) \cdot (5)^6\} =$

A: $(\log_4 3)\,(\log_4 5)^6$ B: $[(\log_4 3)\,(\log_4 5)]^6$

C: $(\log_4 3) + (\log_4 5)^6$ D: $(\log_4 3) + 6\,(\log_4 5)$

E: **NOTA**

15. From the top of a light house 205 ft above the level of the sea at low tide, the angle of depression of a buoy is observed to be 29° 14' at low tide and 28° 12' at high tide. How high is the tide? Round your answer to the nearest tenth .

A: 196.4 ' B: 366.3' C: 8.6' D: 1.8' E: NOTA

16: When a system of linear equations is dependent, the use of Cramer's Rule will result in

A: both the numerator's determinant and the denominator's determinant being equal to zero.
B: The numerator's determinant being equal to zero, but not the denominator's determinant.
C: The denominator's determinant being equal to zero, but not the numerator's determinant.
D: both the numerator's determinant and the denominator's determinant being different from zero.
E: NOTA

17: Two buoys are 64.2 yards apart and a boat is 74.1 yards from the nearer buoy. The angle between the lines from the buoys to the boat is 27° 18'. How far is the boat from the farther buoy?

A: 120.3 yards
B: 120 yards
C: 34.0 yards
D: 31.9 yards
E: NOTA

18: Which choice is the augmented matrix of the following system of equations?

$$\begin{aligned} x - 2y &= 1 \\ y - z &= 5 \\ 2x + 3z &= 1 \end{aligned}$$

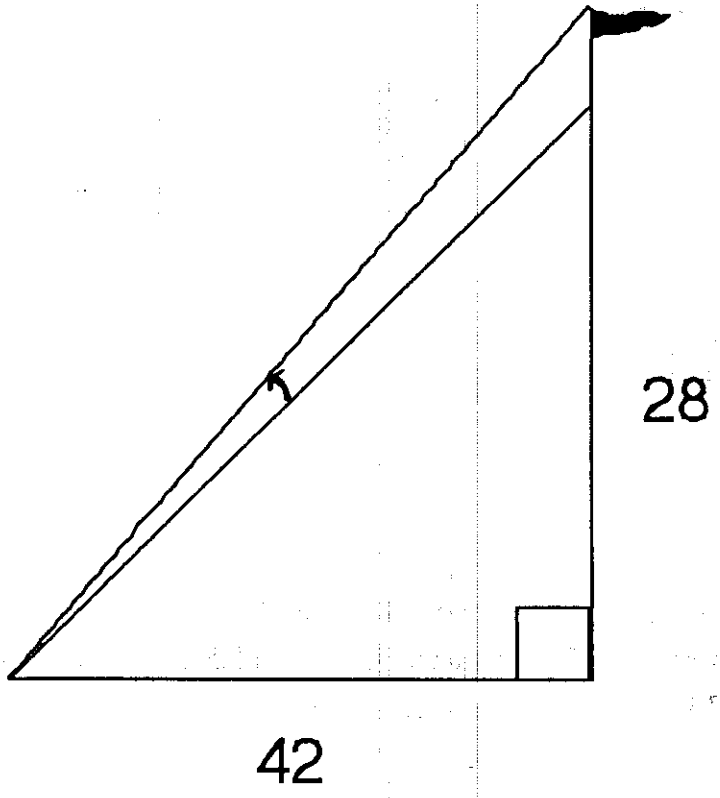
A: $\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 1 & -1 & 5 \\ 2 & 3 & 1 \end{array}\right]$ B: $\left[\begin{array}{cc|c} 0 & -2 & 1 \\ 0 & 0 & 5 \\ 2 & 3 & 1 \end{array}\right]$ C: $\left[\begin{array}{ccc|c} 1 & -2 & 0 & 1 \\ 0 & 1 & -1 & 5 \\ 2 & 0 & 3 & 1 \end{array}\right]$

D: $\left[\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 1 & -1 & -5 \\ 2 & 0 & 3 & -1 \end{array}\right]$ E: NOTA

19: Two stations , A and B, on opposite sides of a mountain ,are both visible from a third station C The distance AC = 11.5 mi., BC = 9.4 mi., and $\angle ACB = 59° 30'$. Find the distance between A and B .

A: 14.9 mi B: 10.5 mi. C: 110.9 mi.
D: 109.7 mi. E: NOTA

20: A building has a flagpole mounted on the top. At a point 42 feet from the building the flagpole subtends an angle whose tangent is 1/3. The building is 28' high, find the length of the flagpole.



A: 26 ' B: 54 ' C: $11\frac{5}{6}$ ' D: 28 ' E: NOTA

21: The sides of a rhombus measure 10 meters and one diagonal is 12 meters. Find the area.

A: $100m^2$ B: $120m^2$ C: $48m^2$ D: $96m^2$ E: NOTA

22: Solve the equation for values of x from 0 to 2π :
cos 2x csc x + csc x + cot x = 0

A: $\pi/2, \quad 3\pi/2$ B: $2\pi/3 \ , \ 4\pi/3$ C: $\pi/2, \quad 2\pi/3$
D: all of the above E: NOTA

23: Given the three sequences defined as $a_n = \frac{1}{n}$, $b_n = \frac{1}{n^3 - 6n^2 + 12n - 6}$,

and $c_n = \begin{cases} -\frac{1}{n} & \text{for n a multiple of 4} \\ \frac{1}{n} & \text{otherwise} \end{cases}$, the set $\left\{1, \frac{1}{2}, \frac{1}{3}, -\frac{1}{4}\right\}$ is the first four terms of:

A: a_n only B: b_n only C: c_n only
D: a_n only and c_n only E: NOTA

24: Which of the following is a harmonic sequence?

A: 2, 4, 6, 8, . . . B: 1, 4, 9, 16, . . . C: 3, 6, 9, 12, . . .
D: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ E: NOTA

25: Suppose $\{ a_n \}$ is convergent and $\{ b_n \}$ is divergent. Which of the following are sometimes convergent ?

i. $\{ a_n + b_n \}$ ii. $\{ a_n \cdot b_n \}$ iii. $\left\{ \frac{a_n}{b_n} \right\}$

A: i only B: ii only C: ii and iii only
D: iii only E: NOTA

26: Find the limit of $\frac{6n^2 - 3n + 8}{2n^2}$ if it exists.

A: $8\frac{1}{2}$ B: it does not exist C: $5\frac{1}{2}$

D: 3 E: NOTA

27: Given $A_t = Pe^{\frac{rt}{100}}$. About how many years would it take to double P dollars if you invest your money at 4 percent and it is compounded continuously? (r = 4)

A: 15 B: 17 C: 20
D: 23 E: NOTA

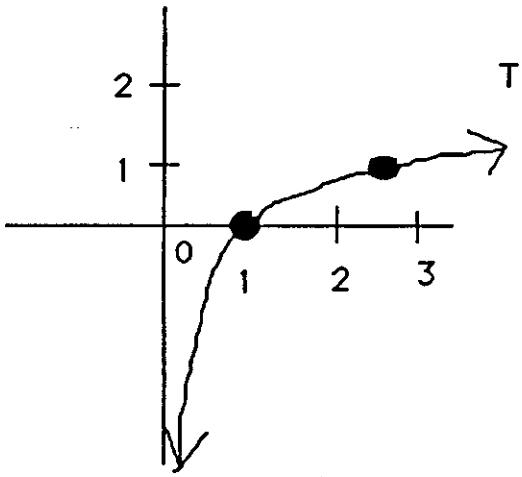
28: An upper bound of the real zeros of $12x^3 - 11x^2 - 7x + 6 = f(x)$ is

A: 2/3 B: -3/4 C: 1
D: 0 E: NOTA

29: Approximate e^{-1} correct to 5 decimal places.

A: .370 B: .36788 C: .36787
D: .36765 E: NOTA

30:



This is the graph of

A: $f(x) = 2^x$ B: $f(x) = e^x$ C: $f(x) = \log x$
D: $f(x) = \ln x$ E: NOTA