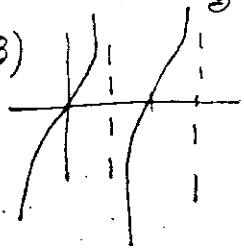
Pre-Calculus Unit Review Solutions

1) $d = \sqrt{x^2+y^2} = \sqrt{x^2+(9x-x^2)}$ The domain of this function is restricted to the x-values between the launch point $(0,0)$ and the point of impact (the other x-intercept) Solving $0 = 9x - x^2 = x(9-x)$ we find the other x intercept to be $(9,0)$. Therefore, $0 \le x \le 9$ so the domain is $[0,9]$ (d)

2) $f(2) = \frac{1}{2}$ $f\left(\frac{1}{2}\right) = 2$ $f(f(2)) \cdot f(f(-3))$

$f(-3) = -\frac{1}{3}$ $f\left(-\frac{1}{3}\right) = -3$ $2 \cdot -3 = -6$ (a)

3) 

i. no ii. no iii. yes iv. no v. no ∴ ii + iii only (c)

iii yes it is non-decreasing

4) $x = 3\cos\theta$ $y = 4\sin\theta$ adding we get $\frac{y^2}{16} + \frac{x^2}{9} = \sin^2\theta + \cos^2\theta$

$\frac{x}{3} = \cos\theta$ $\frac{y}{4} = \sin\theta$ $\frac{x^2}{9} + \frac{y^2}{16} = 1$ ellipse (b)

$\frac{x^2}{9} = \cos^2\theta$ $\frac{y^2}{16} \sin^2\theta$

5) If $\frac{2}{3} + \frac{i\sqrt{5}}{3}$ is a root so is $\frac{2}{3} - \frac{i\sqrt{5}}{3}$ so $x = \frac{2}{3} + \frac{i\sqrt{5}}{3}$, $x = \frac{2}{3} - \frac{i\sqrt{5}}{3}$

$\left(x - \frac{2}{3} - \frac{i\sqrt{5}}{3}\right)\left(x - \frac{2}{3} + \frac{i\sqrt{5}}{3}\right) = 0$ $\left(x - \frac{2}{3}\right)^2 - \frac{5i^2}{9} = 0$ $x^2 - \frac{4}{3}x + \frac{4}{9} + \frac{5}{9} = 0$

$x^2 - \frac{4}{3}x + 1 = 0$ $3x^2 - 4x + 3 = 0$ (c)

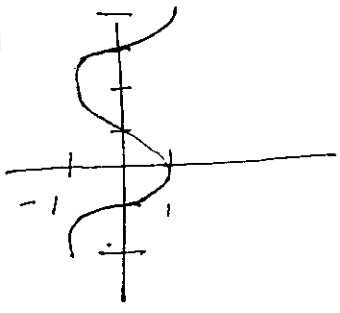
6)

4	1	$0 - k$	8
		4	16 $\quad 64 - 4k$
	1	4	$16 - k$ $\quad 72 - 4k$

$72 - 4k = 0$

$72 = 4k$ (e)

$18 = k$

7) 

∴ $D = [1, -1]$

$R = (-\infty, \infty)$ (d)

Pre-Calculus Individual Test Solutions

8) $\cos^2 x + \sin^2 x = 1$ (a)

9) (b)

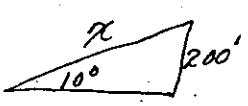
10) $\frac{\cos x + 1}{\sin^3 x} = \frac{\cos x + 1}{\sin x \cdot \sin^2 x} = \frac{\cos x + 1}{\sin x (1 - \cos^2 x)} = \frac{1}{\sin x (1 - \cos x)} =$

$(1 - \cos x)(1 + \cos x)$

$\frac{1}{\sin x} \cdot \frac{1}{(1 - \cos x)} = \frac{\csc x}{1 - \cos x}$ (d)

11) $\tan 165^\circ = -\tan 15^\circ = -\tan \frac{30^\circ}{2} = -\sqrt{\frac{1 - \cos 30}{1 + \cos 30}} = -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}}}$

$-\sqrt{\frac{2 - \sqrt{3}}{2 + \sqrt{3}} \cdot \frac{\sqrt{2 - \sqrt{3}}}{\sqrt{2 - \sqrt{3}}}} = -\frac{(2 - \sqrt{3})}{\sqrt{4 - 3}} = \frac{-2 + \sqrt{3}}{\sqrt{1}} = -2 + \sqrt{3}$ (d)

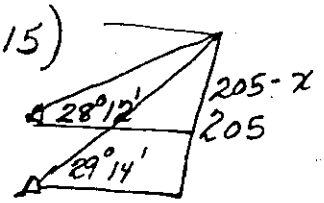
12) 

$\sin 10^\circ = \frac{200'}{x}$ $\quad \frac{30 mi}{hr} = \frac{158,400 ft}{hr} = \frac{2640'}{min} = \frac{44'}{sec}$

$x = 1151.754\ldots$

$1151.754' \div \frac{44'}{sec} = 26.17 sec$ (d)

13) (c) $\quad$ 14) (d)

15) 

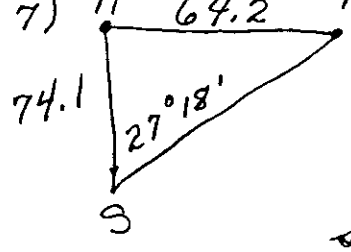
$\tan 28°12' = \frac{205 - x}{y}$ $\quad y = \frac{205 - x}{\tan 28°12'}$ $\quad \frac{205 - x}{\tan 28°12'} = \frac{205}{\tan 29°14'}$

$\tan 29°14' = \frac{205}{y}$ $\quad y = \frac{205}{\tan 29°14'}$

$-.559644851 2\ldots x + 114.727191 45\ldots = 109.9200356\ldots$

$x = 8.58965\ldots \approx 8.6'$ (c)

16) (a)

17) 

$\frac{64.2}{\sin 27°18'} = \frac{74.1}{\sin B}$

$\angle B = 31° 58'$ $\quad \angle A = 120° 44'$

$\frac{a}{\sin 120° 44'} = \frac{64.2}{\sin 27°18'}$

$a = 120.3$ yd. (a)

Pre-Calculus Midwinter Test Review

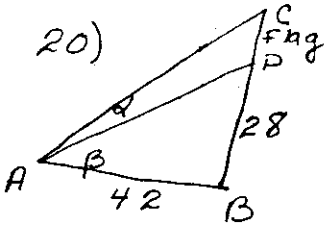
18) (C)

19) $C^2 = 11.5^2 + 9.4^2 - 2(11.5)(9.4)\cos 59.5$

$C^2 = 220.61 - 109.729746\ldots$

$C^2 = 110.88 \quad C = 10.5$ miles (b)

20)



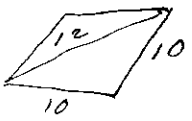
$\tan\beta = \frac{28}{42} \quad \tan\alpha = \frac{1}{3}$

$\angle\beta = 33.69° \quad \angle\alpha = 18.43° \quad \angle A = 52.12°$

$\tan 52.12° = \frac{CB}{42} \quad CB = 53.99$

$CB - 28 = 53.99 - 28 = 25.99 \sim 26'$ (a)

21)



$K = \sqrt{s(s-a)(s-b)(s-c)} \quad s = \frac{32}{2} = 16$

$K = \sqrt{16 \cdot 6 \cdot 6 \cdot 4} = \sqrt{2304} = 48 \times 2 = 96$ (d)

22) $\cos 2x \csc x + \csc x + \cot x = 0$

$(2\cos^2 x - 1)\frac{1}{\sin x} + \frac{1}{\sin x} + \frac{\cos x}{\sin x} = 0 \quad \frac{2\cos^2 x - 1 + 1 + \cos x}{\sin x} = 0$

$\frac{2\cos^2 x + \cos x}{\sin x} = 0 \quad \sin x \neq 0 \quad \cos x(2\cos x + 1) = 0$

$\cos x = 0 \quad \cos x = -\frac{1}{2}$ (d)

$x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \frac{2\pi}{3}, \frac{4\pi}{3}$

23) $a_n = \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ no

$c_n = \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{-1}{4}$

$b_n = \frac{1}{1-6+12-6} = \frac{1}{1}$

$\frac{1}{8-24+24-6} = \frac{1}{2}$

$\frac{1}{27-54+36-6} = \frac{1}{3}$

$\frac{1}{64-96+48-6} = \frac{1}{10}$ no

(C)

24) (d)

25) Example of convergent

ii Let $a_n = \frac{1}{n}$ & $b_n = n$

iii Let $a_n = 2$ & $b_n = n^2$

Example of divergent

Let $a_n = 1$ & $b_n = n$

Let $a_n = 3$ & $b_n = (-1)^n$

(C)

Pre-Calculus Individual Test Solutions

26) $\lim_{n\to\infty} \dfrac{\frac{6n^2}{2n^2} - \frac{3n}{2n^2} + \frac{8}{2n^2}}{\frac{2n^2}{2n^2}} = \lim_{n\to\infty} \dfrac{3 - \frac{3}{2n} + \frac{4}{2n^2}}{1} = 3$ (d)

27) $2P = Pe^{\frac{4r}{100}}$ $\quad \frac{2P}{P} = e^{\frac{4r}{100}}$ $\quad 2 = e^{\frac{4r}{100}}$ $\quad \ln 2 = \frac{4r}{100}\ln e$

$\dfrac{.6931471806\ldots}{4} \cdot 100 = r$ $\quad r = 17.3286\,7951\ldots \sim 17\%$ (b)

28) $\begin{array}{c|cccc} 1 & 12 & -11 & -7 & +6 \\ & & 12 & 1 & -6 \\ \hline & 12 & 1 & -6 & \end{array}$

$12x^2 + x - 6 = 0$

$(4x + 3)(3x - 2) = 0$

$x = -\frac{3}{4} \quad x = \frac{2}{3}$ $\therefore$ (c)

29) $(e \wedge 1)^{-1} = .3678794412\ldots$ $\quad .36788$ (b)

30) $f(x) = \ln(x)$ because $1 = \ln x$

$e = x$ (d)

$x = 2.7\ldots$

Team Solutions

1) the coins are pennies, nickle, dime, quarter + half dollar. number of subsets are $2^5 = 32$ but $\emptyset$ subset does not count as you must have at least 1 coin so 31 different sums can be formed (31)

2) $y = \arcsin -\frac{\sqrt{3}}{2}$ $\quad$ b) no

a) $y = 300° + 360n,\ 240° + 360n$

or

$\frac{5}{3}\pi + 2\pi n,\ \frac{4}{3}\pi + 2\pi n$

Answer Sheet for Test "2/5", 11/24/93

PRE-CALCULUS
REGIONAL COMPETITION

INDIVIDUAL TEST
FEBRUARY 5, 1994

Test Question	Correct Answer
1	D
2	A
3	C
4	B
5	C
6	E
7	D
8	A
9	B
10	D
11	D
12	D
13	C
14	D
15	C
16	A
17	A
18	C
19	B
20	A
21	D
22	D
23	C
24	D
25	C
26	D
27	B
28	C
29	B
30	D