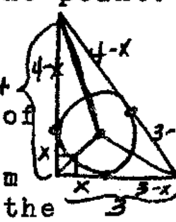
Cypress '95

GEOMETRY TEAM SOLUTIONS FEBRUARY

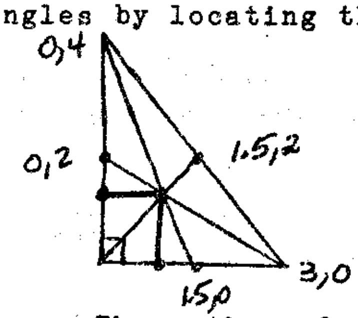
1. A = 100, thus each side = 10. r = 1, thus d = 2, thus 5 across and 5 down, thus 25 circles.

2. Area of each wall is 270; area of each triangle is 105; area of each window is 6; of each door is 19.5. Add up the areas of the rectangles and triangles and subtract the areas of the twelve windows and two doors to get 1389 sq. ft. Multiply by .35 to get $486.15.

3. I. Use the perpendicular bisectors of the sides (any two will do). Note they meet at the midpoint of the hypotenuse.
II. Use the angle bisectors of the triangle to locate the point. Draw the inscribed circle and label the segments of the sides as shown. Since 4-x + 3-x = 5, x = 1 and the coordinates are (1,1).



IV ~~III~~. Draw the altitudes. Note they meet at the vertex of the right angle, (0,0).
III ~~IV~~. The medians meet at a point 2/3 of the distance from each vertex to the midpoint of the opposite side. Draw the medians as shown. Create similar triangles by locating the point 2/3 the distance from (3,0) to (0,0) and drawing a vertical segment; locate the point 2/3 the distance from (0,0) to (0,2) and draw a horizontal segment. The coordinates of the point where these segments meet are (1, 1 1/3).

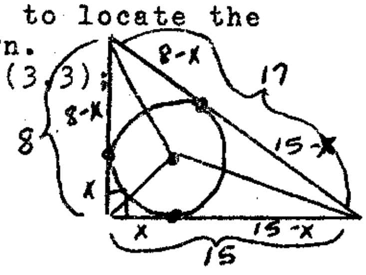


4. Call the original height of the water x. Thus, the volume of the water originally is 8(18) (x). The height of the water after rock is submerged is x + .5. Thus, the volume of the water plus the rock is (8) (18) (x + .5) = 144x + 72. Subtracting the two volumes, the volume of the rock is 72.

5. 5x + 3x + 2x = 20, thus x = 2, so the radii are 10, 16, and 20. Subtract the area of the small sector from the area of the medium sized sector to get the area of the shaded region. $20/360(256\pi)$ minus $20/360\ (100\pi) = (26\pi)/3$.

6. The volume of the sphere = the volume of the cube. Volume of the sphere = $4/3\,\pi(6.4)^3$. Compute the volume and take the cube root of the answer to find the edge of the cube since $V = e^3$. The volume will vary depending on the approximation of π used, but when rounded to the nearest tenth, the anser is 10.3.

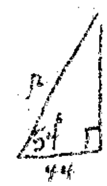
7. Draw the inscribed circle and the angle bisectors to locate the center as shown. Label the side segments as shown. Since 8-x + 15 - x = 17, x = 3; the center is at (3,3); the radius = 3; thus the area = 9π.

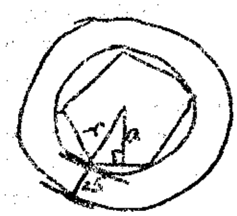


Use formulas for chords intersecting in a circle to find EB. (4)(12) = 6x. x = 8. Then drop altitudes in each of the triangles to form 30-60-90 triangles. Use 30-60-90 relationships to find the lengths needed to find the areas. Area of EBD = $24\sqrt{3}$ and area of CAE = $6\sqrt{3}$. Thus the sum is $30\sqrt{3}$

9. Draw radii (perpendicular to tangents). Draw a segment parallel to to tangent from point of tangency of smaller circle. Work with the triangle formed (note 30-60-90 relationships since hypotenuse is twice a leg). Add the two tangents and the two arcs involved (note the arc for the larger circle is 240 degrees and thus 2/3 of the circumference and the arc for the smaller circle is 120 degrees and thus 1/3 of the circumference). $2(12\sqrt{3}) + 20\pi + 2\pi = 24\sqrt{3} + 22\pi$.

10. Use regular polygon facts, then rt. triangle trig to get the radius of the pentagon. Each angle of the pentagon = (5-2)180 ÷ 5 = 108 degrees. See figure.
$\cos 54^\circ = 44/r$. $r = 44/\cos 54^\circ$.
The radius of the circular fence = r + 25. With varying values of π, one gets a circumference of between 627 feet and 628 feet; since the fencing is purchased by the foot, 628 feet must be purchased.





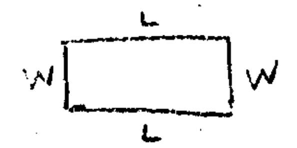
11. A little know theorem states that CD: DA ::AE:EF. Thus AE = 5. To find AE without this theorem, set up a system of equations using secant/tangent theorems and the pythagorean theorem: Refer to figure.
$(10)(2) = t^2$, thus $t^2 = 20$. Also, $11(11+x) = s^2$, thus $121 + 11x = s^2$
Also, $t^2 + d^2 = 10^2$ and $s^2 + d^2 = (11+x)^2$
By substitution, $20 + d^2 = 100$ thus $d^2 = 80$
Substituting again, $s^2 + 80 = (11 + x)^2$
and $121 + 11x + 80 = (11+x)^2$
Solving for x, x = -16 which we reject and x = 5. Thus, AE = 5.

12. Refer to figure with equal segments and angles labeled. Note many similar triangles. Using similar triangles FBE and ADG, $12/a = a/8$ thus $a = \sqrt{96} = 4\sqrt{6}$. Use area of trapezoid formula: $A = 1/2\ (4\sqrt{6})\ (4\sqrt{6} + 4\sqrt{6} + 20)$
Thus, the area is $96 + 40\sqrt{6}$.

13. Label equal angles and note similar triangles PST and RQP. Thus. $10/x = 3/5$. Thus $x = 50/3$ (answer to I). Use Pythagorean theorem to find PR in triangle RQP. Then subtract PS to get SR.
$PR^2 = 5^2 + (50/3)^2$ leads to $PR = \sqrt{\frac{2725}{9}} = (5\sqrt{109})/3$

Thus $SR = (5\sqrt{109})/3 - 3 = (5\sqrt{109} - 9)/3$ (answer to II)
To find area of trapezoid PQRT, use:
$A = 1/2(50/3)\ (5 + \sqrt{109}) = (250 + 50\sqrt{109})/6 = (125 + 25\sqrt{109})/3$
(Answer to III)

14. This is the Golden Rectangle. Refer to figure. $(L + W)/(L) = L/W$. To find the ratio of L to W, Let $W = 1$. Thus $(1 + L)/L = L/1$. Solving for L, $L = (1 \pm 5)/2$. Disregard the negative solution, so $L = (1+\sqrt{5})/2$. Thus $L/W = (1 + \sqrt{5})/2$.



15. Completing the square in x and in y and balancing the equation, we obtain $x^2 + 6x + 9 + y^2 - 4y + 4 = 12 + 9 + 4$
Factoring, we obtain $(x + 3)^2 + (y - 2)^2 = 25$
Thus, the center is at $(-3,2)$ and thr radius $= \sqrt{25} = 5$.