

Geometry Individual Test

January Regional competition 2005

Solutions to individual test

- D 1. $2x + 3x + 4x = 180, x = 20$.
angles are 40, 60, 80. Smallest exterior angle is the supplement of the largest interior angle which would be 100.
- C 2. Ratio of sides is 14:28 or 1:2.
Ratio of perimeters is 1:2.
Perimeter of ABC=29,
 $\frac{1}{2} = \frac{29}{x}, x=58$.
- B 3. Since lines are parallel, $\angle AGH \cong \angle GHD$
Because they're alternate interior angles.
Therefore, $3x - 10 = 80, x = 30, 2x = 60$.
- C 4. $\triangle ADE \sim \triangle ABC$ by AA. Ratio of sides is 1:2. Let $AE=x, \frac{x}{6} = \frac{1}{2}, x = 3 = AE, EC=3$.
 $\frac{DE}{8} = \frac{1}{2}, DE=4$. Since the triangle is right use Pythag to find $AB=10, AD=BD=5$.
Perimeter of trapezoid = $3+8+5+4=20$.
- D 5. $\angle A$ and $\angle B$ are supplementary because they are consecutive angles.
 $3x - 1 + 2x + 11 = 180, x=34$.
 $\angle C \cong \angle A, m\angle A = 101$
- A 6. $m\angle B = x, m\angle A = 2x, m\angle C = 3x + 6$,
 $6x + 6 = 180, x = 29$
- C 7. $AD : ED = 2 : 1$, since $AD=18, ED=6$.
Use Pythag to find $EC=10$
- C 8. $40 + x + 20 + 30 = 180, x = 90$
- C 9. The width is 15. It's a Pythag triple, so Area is $15 \times 8 = 120$.
- C 10. Area of original is 100π . Radius Decreased by 2 is 8 making area 64π .
Decrease in area is 36π .
 $36\pi : 100\pi = 36\%$ decrease.
- D 11. $\angle FDP \cong \angle EBP$ - alternate interiors
 $\angle DPF \cong \angle BPE$ - vertical angles
 $\triangle DPF \sim \triangle BPE$ by AA. Let $DC=4x$, which makes $DF=2x$. Since $\overline{DC} \cong \overline{AB}$, $AB=4x$ and since $AE=1/4 AB, AE=x$ and $EB=3x$. $DF:BE=DP:BP=2:3$.
- A 12. $180 - x = 2(180 - (90 - x)) - 78, x = 26$
- A 13. Using Pythag on $\triangle ABC, AC=21$. Using Pythag on $\triangle ABD, AD=15$. $AC-AD=6$
- D 14. Let $AM = x, FM = FA = 3x$. Since Perimeter is 42, $7x = 42, x=6$. $FA=18$
- A 15. $64 > 36 + 9$, triangle is obtuse.
- B 16. Use vertical angles for $\angle 1$, then the 3 Angles are supplementary which makes $m\angle 1 = 180 - a - b$
- A 17. I always, II never, III sometimes
- D 18. $m\angle MPN = m\angle MNP = 50$,
 $m\angle PMN = 80, m\angle TMN = 50$ by alternate interior angles.
 $m\angle TMP = m\angle PMN + m\angle TMN = 80 + 50 = 130$.
- A 19. Draw the altitude from point A and let this be point E.. This is also 6. Since $m\angle D = 30, AD=12$ and $DE=6\sqrt{3}$. $\sqrt{192} = 8\sqrt{3}$. So $DC=14\sqrt{3}$.
- B 20. Let $BD=x, AB=x+4$. In $\triangle CBD, BD=x, BC=x\sqrt{3}, CD=2x$. Using $\triangle ABC$, Since $BC=x\sqrt{3}, AB=3x$. We have two expressions for AB: $3x = x + 4, x = 2$, $AB=6, CB=2\sqrt{3}$.
- A 21. $m\angle CEF = 30, m\angle BEF = 50, 50-30=20$.

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- B 22. $\triangle ABC$ is right making $AC=10$,
 $AB = 5\sqrt{3}$. $\triangle BDC$ is equilateral so
 all sides are 5. Perimeter of $\triangle ADB$ is
 $AD+BD+AB=5+5+5\sqrt{3} = 10+5\sqrt{3}$
- C 23. Draw a right triangle. Sides are 5 and 9.
 Find the hypotenuse by using Pythag,
 $c^2 = 25 + 81, = \sqrt{106}$. $c = 10.3$
- C 24. $2l + 2w = 1.5lw$, substituting 4 for l ,
 $8 + 2w = 6w, w = 2$, area = 8.
- B 25. There are 3 possibilities for the
 parallelogram. Sides of 3 and 4 (which
 is a rectangle), sides of 4 and 5, sides of
 3 and 5 – both of which are NOT
 rectangles. The perimeters of all 3 will
 be different. However, the areas of all
 3 will be the same and will be 12.
- C 26.
$$\frac{(n-2)180}{n} = \frac{11(180)}{13} = \frac{1980}{13}$$
- D 27. Circumference = 60π cm,

$$\frac{120 \cdot 100 \cdot 1000}{60\pi} \approx 63662$$
- D 28. Since \overline{SU} bisects the angle, the two
 smaller angles are congruent.
 $2x = 20 = 8x - 14, x = 4$,
 $m\angle TSU = 18$
- C 29. The triangles are similar since the lines
 are parallel. $\frac{12}{17} = \frac{4}{x}, x = \frac{17}{3}$
- D 30. One way to solve this is to find the area
 using the legs, then finding the area
 using the hypotenuse and its altitude.
 This gives the equation $\frac{1}{2}xy = \frac{1}{2}hz$.
 Solving for h gives $\frac{xy}{z}$.

Geometry January Regional 2005 Individual Answers

1. D
2. C
3. B
4. C
5. D
6. A
7. C
8. C
9. C
10. C
11. D
12. A
13. A
14. D
15. A
16. B
17. A
18. D
19. A
20. B
21. A
22. B
23. C
24. C
25. B
26. C
27. D
28. D
29. C
30. D