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Answers to January Regional Competition – Geometry Division

Individual

1	C
2	D
3	D
4	A
5	B
6	C
7	B
8	B
9	C
10	E
11	C
12	B
13	C
14	D
15	D
16	C
17	C
18	C
19	A
20	B
21	C
22	C
23	C
24	C
25	D
26	B
27	A
28	A
29	A
30	B

Team

1	$\frac{19}{5}+\frac{8\pi}{3}$
2	$4\sqrt{3}$
3	480
4	$\frac{-17}{7}$
5	80
6	5.5
7	1.5
8	2
9	$\frac{27\sqrt{3}}{2}$
10	2172.4
11	28
12	242.0
13	90^{O}
14	1024π
15	infinite

Individual Solutions – January 2001 – FAMAT Regional – Geometry Division

1) First you must find the angle between the hands. At 2:45, the minute hand has moved 0.75 of 360, or 270°. You must subtract from that the degrees passed by the hour hand: 30° each hour plus 0.75(30) or 22.5. Thus 270 –30 –30 –22.5 = 187.5°. However, the question asks for the minor arc, so the angle in question is 360-187.5 or 172.5°. The arc length thus would be $\frac{172.5}{360}(2\pi(4))$ or $\frac{23\pi}{6}$, or C.
2) Since they are supplementary their sum is 180°, so 4x + 2 + 6x + 3 = 180, so $x = 35/2$. Plug that back into the expressions to find the angles measure 72 and 108. 108:72 = 3:2, or see that the two angles have a common factor of (2x +1), which leaves the ratio of 3:2, choice D.
3) Use 30-60-90 properties to find the side length of the hexagon. The black triangles have a longer base of length 3.5, shorter base then of $\frac{7\sqrt{3}}{6}$. Pythagorean theorm shows the hypotenuse will be $\frac{7\sqrt{3}}{3}$, this also serves as the side of the hexagon. Thus $A_{hexagon} = \frac{6s^2\sqrt{3}}{4}$, plug in "s" and find that $A_{hexagon} = \frac{49\sqrt{3}}{2}$. So the area of the shaded region is $49 - \frac{49\sqrt{3}}{2}$, or D.
4) I & II only, III is only true if they are perpendicular to the transversal, Thus A.
5) Apply the slope formula, $\frac{y2-y1}{x2-x1} = \frac{1-7}{1-3} = \frac{-6}{-2} = 3$, B.
6) Use 30-60-90 triangle properties to find that the shorter leg of the black triangle is $2\sqrt{3}$. Thus $\frac{1}{2}(2\sqrt{3})(6) = 6\sqrt{3}$, or C.
7) $A_{rhombus} = \frac{1}{2}d1d2 = (0.5)(10)(5) = 25$, or B.
8) $A_{total} - A_{top\ pyramid} = A_{bottom\ section}$. Thus $\left(\frac{1}{3}\right)(6^2)(10) - \left(\frac{1}{3}\right)(5^2)(10-x) = 70$, where x is the distance between the bottom base and the plane. Rewrite as $120 - \frac{250-25x}{3} = 70$. Use algebra to solve and find x = 8, so B.
9) The ratio of the area is the square of the ratio of the lengths. Thus $(4:7)^2 = 16:49$, or C.
10) None of these, skew lines do not intersect at all, therefore E.
11) Work backward from the cylinder volume to find that $720\pi = \pi r^2(20)$, so the radius of the cylinder is 6. Thus the largest sphere can only have a radius of 6, so $\frac{4}{3}\pi(6)^3 = 288\pi$, C.
12) The radius of the resulting cylinder would be 5, with a height of 6. Thus $\pi r^2 h = \pi(5^2)(6) = 150\pi$, B.
13) Draw an imaginary radius from the center to point A. This is 5 units long and AB is $4\sqrt{6}$ (given). Use Pythagorean theorem to find that center to B is 11. Subtract 5 (the radius) to find that BC = 6, or C.
14) Use 80° because of the central/inscribed relationship. Thus $\frac{80}{360}\pi 7^2 = \frac{98\pi}{9}$, or D.
15) Draw a diagram showing the points and make a substitution that AB = 2 (or any number). Then work backwards defining the lengths of the segments and test them against the I, II, and III conditions. If you chose AB = 2, then I) becomes 2 + 2 = 4 (true), II) 2 + 3(2) = 8, III) 16 – 4 = 12, so all are true, D.
16) The third side must be between 7 (12 minus 5) and 17 (12 plus 5), not inclusive. Only C can be used, C.
17) Set the expressions each equal to each other, 3x + 2 = 4x – 7, 3x + 2 = 2x + 10, and 4x –7 = 2x + 10. Solve each of these for "x" since these will set two sides equal. These values {8, 8.5, 9} are choice C.
18) Use 30-60-90 properties to find the side length of the hexagon is $\frac{2r}{\sqrt{3}}$. Apply $\frac{6s^2\sqrt{3}}{4}$ to find that the area is $2\sqrt{3}r^2$ or C.

19) Use a series of triangle ratios. $\frac{z}{12} = \frac{12}{24}$, so $z = 6$. $y^2 = 12^2 + 24^2$, so $y = 12\sqrt{5}$. Finally, $x^2 + 720 = (24+6)^2$. So $x = 6\sqrt{5}$. x + y + z thus is $18\sqrt{5} + 6$, or A.

20) First find the volume of the cone, $\frac{1}{3}\pi 6^2(12)$ or 144π. Once in gaseous form the volume will be $(6/5)(144\pi)$ or $\frac{864\pi}{5}$. So the minimum radius is $\frac{864\pi}{5} = \frac{4}{3}\pi r^3$, solve to find that $r = 5.06$, round to the nearest inch to get 5, or choice B.

21) Work backward from $\frac{27\sqrt{3}}{2} = \frac{6s^2\sqrt{3}}{4}$ to find that s = 3 for the hexagon. Thus, the sides of the triangle are 9 units. Apply the equilateral triangle area formula, $\frac{s^2\sqrt{3}}{4}$, to find that the area is thus $\frac{81\sqrt{3}}{4}$, C.

22) Given the area, find the radius of the circumscribed circle to be 7. This serves as the hypotenuse of a small 30-60-90 triangle inside the black triangle. Use 30-60-90 properties to find that the side length of the triangle is $7\sqrt{3}$. Apply the equilateral triangle area formula to find that the area of the black triangle is $\frac{147\sqrt{3}}{4}$, or C.

23) The two sides given dictate that the third side is 12. Thus the rhombus has diagonals of length 10 and 24. Apply the rhombus area formula, $\frac{1}{2}d1d2$, or (0.5)(10)(24) = 120, or C.

24) Remember that (AE)(EC) = (BE)(DB) for that given circle and the points. So (4)(8) = (2)(DB-2). Solve for BD, to find that BD is 18, or C.

25) The only point that would not provide a triangle when joined with the two given points is one that lies on the line segment created which would form three collinear points. Check the slope of the two given points and then compare those with the slopes that would be formed with each of the choices. Thus, only (6, 13) is on that line, so choice D is correct.

26) Work backwards from the volume to find the radius, 6. The great circle is the largest circle around the sphere (such as the equator around the earth). Thus the circumference would be $2\pi r$, or $(2)(\pi)(6)$, or 12π, choice B.

27) First find the area of the triangle using the equilateral triangle area formula: $\frac{s^2\sqrt{3}}{4}$ which becomes $\frac{15^2\sqrt{3}}{4}$ or $\frac{225\sqrt{3}}{4}$. Then you have to subtract out the sector covered by the circle, since it is an equalateral triangle, it's a 60° sector, so $\frac{60}{360}\pi r^2$ or $\frac{27\pi}{2}$. So the final answer is $\frac{225\sqrt{3}}{4} - \frac{27\pi}{2}$, or A.

28) Average the x- and y- coordinates (what the midpoint formula does), to find that $x_{mid} = 2$, and $y_{mid} = 9$, so (2, 9) is correct, A.

29) Parallel lines have equal slopes, so solve the first equation for "y=" and find the slope to be –0.75. Use the point-slope formula to find that $y - 5 = \frac{-3}{4}(x-2)$. Rewrite as 3x + 4y = 26, or A.

30) Use Heron's Formula ($A = \sqrt{s(s-a)(s-b)(s-c)}$) where s = the semi perimeter, and "a", "b", and "c" represent the lengths of the sides. Thus $\frac{21\sqrt{11}}{4}$ or B.