January Regional Calculus Answers

Individual

1. B
2. C
3. E
4. C
5. C
6. C
7. E
8. B
9. D
10. D
11. B
12. E
13. C
14. D
15. C
16. C
17. A
18. A
19. A
20. C
21. A
22. D
23. A
24. D
25. D
26. C
27. E
28. B
29. A
30. B

Team

1. 1
2. $64\pi^3$
3. $12-2\sqrt{3}$
4. A, B, and G
5. $-\frac{1}{5}$
6. 0.041
7. $\frac{3\sqrt{6}}{8}$
8. 34
9. -4
10. 224
11. $\left(\frac{1}{2},\frac{3}{4}\right)$
12. $\frac{2}{\pi}$
13. $1+e$
14. -2π
15. $4\sqrt{3}$

1. $A=0$. $B=0$. $C=0$. $D=1$. $A+B+C+D=\mathbf{1}$.

2. $x^3y'+3x^2y+y'\sin(x)+y\cos(x)+3y^2y'=0$. $y'=-\dfrac{3x^2y+y\cos(x)}{x^3+\sin(x)+3y^2}$.

$y'(0,2)=-\dfrac{1}{6}$. The slope of the normal line is therefore 6. $\lim\limits_{x\to\sqrt{\pi}}\dfrac{\sin(x^2)}{x-\sqrt{\pi}}=f'(\sqrt{\pi})$ where $f(x)=\sin(x^2)$. $f'(x)=2x\cos(x^2)$. $f'(\sqrt{\pi})=-2\sqrt{\pi}$. $(-2\sqrt{\pi})^6=\mathbf{64\pi^3}$.

3. Let the length of the rectangle be x and the height y. Then $P=3x+2y$ and $A=xy+\dfrac{x^2\sqrt{3}}{4}=6-\sqrt{3}$. $y=\dfrac{6-\sqrt{3}}{x}-\dfrac{x\sqrt{3}}{4}$. $P=\dfrac{12-2\sqrt{3}}{x}+\left(\dfrac{6-\sqrt{3}}{2}\right)x$.

$P'=-\dfrac{12-2\sqrt{3}}{x^2}+\dfrac{6-\sqrt{3}}{2}=0$. $x^2=4$. $x=2$ and therefore $P=\mathbf{12-2\sqrt{3}}$.

4. A is true because $f(4)=0$. B is true because $f'(2)=0$ and $f''(2)<0$. C is false, with $f(x)=\dfrac{x^4}{2}-27x^2+92x-64$ as an example. D is false because $f'(4)\neq0$. E is false because $f'(1)>0$. Since f is twice differentiable, $f'(x)$ is continuous on $[3,4]$ and differentiable on $(3,4)$. Since $f'(3)<0$ and $f'(4)>0$, there exists a $c\in(3,4)$ such that $f'(c)=0$ by Rolle's Theorem. Since $f'(2)=0$, f has at least two critical points and therefore F is false. G is true since f and $f'(x)$ are continuous, f is increasing at $x=4$, and $f(4)=0$, which implies that there exists $d\in(3,4)$ such that $f(d)<0$. **A, B, and G**

5. $\lim\limits_{x\to a}\dfrac{x^2+2x+1}{x-3}=\dfrac{a^2+2a+1}{a-3}=a$ as long as $a\neq3$. $a^2+2a+1=a^2-3a$. $a=\mathbf{-\dfrac{1}{5}}$.

6. $r=\dfrac{C}{2\pi}$. $V=\dfrac{4}{3}\pi r^3=\dfrac{4}{3}\pi\left(\dfrac{C}{2\pi}\right)^3=\dfrac{C^3}{6\pi^2}$. $dV=\dfrac{C^2}{2\pi^2}dC$.

relative error $=\dfrac{dV}{V}=\dfrac{\frac{C^2}{2\pi^2}}{\frac{C^3}{6\pi^2}}=\dfrac{3\,dC}{C}=\dfrac{3}{73}\approx\mathbf{.041}$.

7. $A=x\left(1+\sqrt{1-2x^2}\right)$. $1+\sqrt{1-2x^2}-\dfrac{2x^2}{\sqrt{1-2x^2}}=0$. $\sqrt{1-2x^2}+1-4x^2=0$.

$1-2x^2=16x^4-8x^2+1$. $x^2(16x^2-6)=0$. $x=0$ or $x=\pm\dfrac{\sqrt{6}}{4}$. Obviously $x=0$ is the minimum and $x=\pm\dfrac{\sqrt{6}}{4}$ are both maximums. $A=\dfrac{\sqrt{6}}{4}\left(1+\sqrt{1-2\left(\dfrac{\sqrt{6}}{4}\right)^2}\right)=\mathbf{\dfrac{3\sqrt{6}}{8}}$.

8. Since f is odd, $f(-x)=-f(x)$. Differentiating both sides gives $-f'(-x)=-f'(x)$, or $f'(-x)=f'(x)$. Therefore $f'(x)$ is even. Similarly $f''(x)$ is odd. Neither $f'(x)$ nor $f''(x)$ need be one-to-one. An example of this is $f(x)=\cos(x)$. Since f is twice differentiable, $f'(x)$ is continuous of the closed interval $[-10,10]$, and therefore it has both a maximum value and a minimum value on that interval. $0+15+0+0-2+21=\mathbf{34}$.

9. $D_x\left[g\left(\frac{1}{2}f(x)g'(x)\right)\right]=g'\left(\frac{1}{2}f(x)g'(x)\right)\cdot\frac{1}{2}\left(f'(x)g'(x)+f(x)g''(x)\right)$.

$g'\left(\frac{1}{2}f(2)g'(2)\right)\cdot\frac{1}{2}\left(f'(2)g'(2)+f(2)g''(2)\right)=g'(2)\cdot\frac{1}{2}\cdot-6=\frac{4}{3}\cdot\frac{1}{2}\cdot-6=\mathbf{-4}$.

10. Let x be the number of feet of wire that runs at the edge of the road. Then the number of feet of wire running under the road is $\sqrt{20^2+(48-x)^2}$. The total cost is then $C=5\sqrt{x^2-96x+2704}+3x$. $C'=\frac{5(2x-96)}{2\sqrt{x^2-96x+2704}}+3=0$.
$5x-240=-3\sqrt{x^2-96x+2704}$. $25x^2-2400x+57600=9x^2-864x+24336$.
$x^2-96x+2079=0$. $x=33$ or $x=63$. Obviously the only possible solution is $x=33$.
$C=5\sqrt{33^2-96\cdot33+2704}+3\cdot33=\mathbf{224}$.

11. $d=\sqrt{(x-1)^2+\left(x^2+\frac{1}{2}-\frac{1}{4}\right)^2}=\sqrt{x^4+\frac{3x^2}{2}-2x+\frac{17}{16}}$. $d'=\frac{4x^3+3x-2}{2\sqrt{x^4+\frac{3x^2}{2}-2x+\frac{17}{16}}}=0$.

$x=\frac{1}{2}$. $y=\left(\frac{1}{2}\right)^2+\frac{1}{2}=\frac{3}{4}$. The point closest to the graph is $\left(\mathbf{\frac{1}{2},\frac{3}{4}}\right)$.

12. This Riemann sum is equivalent to $\int_0^1\cos\left(\frac{\pi}{2}x\right)dx=\frac{2}{\pi}\sin\left(\frac{\pi}{2}x\right)\Big|_0^1=\mathbf{\frac{2}{\pi}}$.

13. $f'(x)=xe^x+e^x$, $f''(x)=xe^x+2e^x$, $f'''(x)=xe^x+3e^x$, …, $f^{(n)}(x)=xe^x+ne^x$.
$A=2005$. $B=2007e$. $C=2006e$. $D=2004$. $A+B-C-D=\mathbf{1+e}$.

14. $f'(x)=\cos(x)-x\sin(x)$. $f'(\pi)=-1$. The slope of the normal line is therefore 1.
$1(x-\pi)-\pi=y$. $A=1$ and $B=-2\pi$. $1\cdot-2\pi=\mathbf{-2\pi}$.

15. $d=s\sqrt{2}$. $dd=\sqrt{2}\,ds$. $ds=\frac{\sqrt{2}}{2}dd$. $A=s^2$.
$dA=2s\,ds=s\sqrt{2}\,dd=2\cdot\sqrt{2}\cdot\sqrt{6}=\mathbf{4\sqrt{3}}$.