

CALCULUS TEAM QUESTIONS

January

1. Find the equation of the line tangent to the graph of $f(x) = 2x^3 - 7x + 1$ at $(2, 3)$.

2. $A = \lim_{x \rightarrow 2} \frac{x+2}{x^2+4}$

$$B = \lim_{x \rightarrow 3} f(x) \text{ where } f(x) = \begin{cases} x^2 - 4 & \text{for } x \leq 3 \\ \frac{x+2}{x-2} & \text{for } x > 3 \end{cases}$$

$$C = \lim_{x \rightarrow \infty} \frac{4x^3 + 8x^2 - 1}{\sqrt{x^6 + 1}}$$

Find $4A + 2B - 3C$.

3. Differentiate $f(x) = x^3 e^x$.

4. The population of a city at time t years is given by $P(t) = 50,000(\sqrt{t} + 3)$. At what rate is the population growing when $t = 4$?

5. An open cylindrical bucket is to be built with a volume of 1 cubic foot. Find the dimensions that minimize the area of material used in its construction. Give radius and height exact.

6. A line is tangent to $y = x^3$ at $(2, 8)$. At what point does it again meet the curve?

7. On the curve $x^2 + 2xy - y^2 + 2y + 4 = 0$, find the coordinates of every other point where the tangent line is parallel to the tangent line at $(-2, 2)$.

8. Evaluate $\int_0^{\pi} (e^x + 2 \sin x) dx$. Give answer exact.

9. Find the coordinates of the extrema of $f(x) = \sin^2 x + 2 \cos x$ on $\left[0, \frac{3\pi}{2}\right]$.

10. Evaluate $\int_0^{\frac{1}{2}} \left(\frac{5}{\sqrt{1-x^2}} \right) dx$. Give answer exact.

11. Each side of a square baseball diamond is 90 ft. long. If a ball is hit down the third base line at a speed of 100 ft/s, how fast is the distance from the ball to first base changing when the ball is halfway to third base? Give answer exact.

12. $A = \int_0^2 (2x^7 - x^3 + 5) dx$

$$B = \int_{-8}^{-1} \frac{1}{\sqrt[3]{x^2}} dx$$

$$C = \int_1^4 (2x^{5/2} + x^3) dx$$

Find $(A-14B)C$.

13. If $f'(x) = e^x + 1 + 2 \sin x$ and $f(0) = 4$, find $f(x)$.

14. Find the area of the region bounded by the graphs of

$$f(x) = x^3 - 3x + 2 \text{ and } g(x) = x + 2$$

15. Differentiate $f(x) = x^x$.