

Team Questions List for Sponsor Pack January Regional MAØ Calculus Test 1997

- 1) An isosceles triangle is inscribed inside a parabola $y = x^2$ with one vertex of the triangle at the vertex of the parabola and the other two vertices lying on the parabola and also on a horizontal line (perpendicular to the parabola's axis). How fast is the area of the triangle changing when its height is 2 units if the height is increasing at 6 units per second? Give an exact answer.

2) $A = D_x \left[\frac{1}{x^3 - 6} \right]$ when $x = 2$.

$B = D_x [3x^4 - 12x^3 + 3x^2 - 18x + 5]$ when $x = 3$.

$C = D_x \left[\sqrt{(x^2 + 1)^3 + 1} \right]$ when $x = 1$.

What is $1A + 2B + 3C$?

- 3) The position of the shadow of a cog moving in a machine can be described by the function $s(t) = 2 \sin(t) + \cos(2t)$.
What is the position of the shadow the first time its velocity is zero?

- 4) A cylindrical grain silo is as high as it is in diameter, and it has half a sphere on top. If the circumference is calculated at 60 feet, with a possible error of one inch, use differentials to calculate the error in the volume in cubic feet to three decimal places.

5) Let $f(x) = \int_1^x \frac{e^{-t}}{t^2} dt + \frac{x^2 - 2x + 4}{x - 2} - x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + \sum_{i=0}^{\infty} \frac{x}{\pi^i}$.

Find $\lim_{x \rightarrow \infty} f'(x)$.

- 6) Find the line perpendicular to the line tangent to the graph $y = 5^{x^2} + \operatorname{arccot}(2x)$ at $x=0$ through $(1,1)$ in point-slope form based on this point.

7) Find the domain of $q(w) = \frac{1}{\sqrt{-w^2 + 4w - 1}}$.

- 8) Given that $f'(x) = 2xy$ and that $y = e^2$ when $x = 1$, find y when $x = 0$.

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- 9) What is $\frac{97}{\int_0^1 \sqrt{x}(\sqrt[3]{x} + 1)^2 dx}$ (besides being a very important number)?
- 10) Find a and b so that the function $f(x) = ax^3 + bx^2 - 2x + 2$ has an inflection point at $(-1, 8)$.
- 11) Let (A, B) and (C, D) be the points where two distinct, non-vertical lines intersect normal to the curve $y = x^2 - 2$. Both lines also pass through $(0, 2)$. What is AC ?
- 12) If $(xy)^2 - 2y = 0$, $y \neq 0$, then what is $\lim_{x \rightarrow 1} \frac{dy}{dx}$?
- 13) Find $f'(2)$ if $f(x) = e^{2x} - 3\sqrt[3]{x^2 + 4} + \ln(2x) + \frac{\arcsin(2x - 4)}{4}$
- 14) A young woman, after having her heart broken, throws her gold locket straight up off a 72 ft cliff at an initial vertical velocity of 30 ft/s.
- A = The maximum height the locket reaches
B = The time required for the locket to hit the ground
C = The vertical speed of the locket at point A
D = The vertical speed of the locket when it is 2 feet above the base of the cliff.
- Find $\frac{AC}{BD}$.
- 15) A baseball diamond has the shape of a square with sides 90 feet long. Bases are numbered counterclockwise as Home, 1st, 2nd, and 3rd. A player runs from 1st to 2nd at 24 ft/s. At what rate is the player's distance from 3rd base changing when the player is 20 feet away from 2nd? Answer to 2 decimal places and indicate increasing or decreasing.