CALCULUS BOWL FANAT REGIONAL JANUARY 11, [redacted] 1992

1. $\frac{3}{2} + \frac{\pi}{2} - 16e^3$ or $\frac{3 + \pi - 32e^3}{2}$

2. $\frac{-2\sqrt{e}}{e}$

3. $\frac{3 + \sqrt{2}}{2}$

4. $-2048\cos(2x) + 2048\sin(2x)$

5. $\frac{e + 1}{8\ln(3)}$

6. $x + y - 2 = 0$

7. $(-3,-2)$ or $-3 < x < -2$

8. $\frac{3\pi}{16}$

9. -12

10. $48\sqrt{3}$

11. $y = 1$ or $y - 1 = 0$

12. $\ln(4)$

13. 15

14. $8 + 2\sqrt{3}$

15. $\frac{-1}{2}$

Calculus Bowl FAMAT Regional 1/11/92

1. $f'(x) = \frac{4x}{2\sqrt{2x^2+2}} - \frac{2(x+1)-2y}{(x+1)^2} + \frac{2x}{1+x^2} + 2\arctan x - 16xe^{2x^2+1}$

$f'(1) = 1 - \frac{1}{2} + 1 + \frac{\pi}{2} - 16e^3 = \frac{3}{2} + \frac{\pi}{2} - 16e^3$

2. $x(2yy') + y^2 = 2xe \quad 2\sqrt{e}\,y' + e = 2e \quad y' = \frac{e}{2\sqrt{e}} \quad m_{norm} = \frac{-2\sqrt{e}}{e}$

3. $x(\frac{\pi}{4}) = \frac{1}{2}\sin(\frac{\pi}{4}) + \cos(\frac{\pi}{2}) = \frac{\sqrt{2}}{4} \quad v(t) = \frac{1}{2}\cos t - 2\sin(2t) \quad v(\frac{\pi}{4}) = \frac{\sqrt{2}}{4} - 2$

$a(t) = -\frac{1}{2}\sin t - 4\cos(2t) \quad a(\frac{\pi}{2}) = -\frac{1}{2} + 4 \quad A+B+C = \frac{\sqrt{2}}{2} + \frac{3}{2}$

4. $f'(x) = 2\cos(2x) - 2\sin(2x) \quad f'' = -4\sin(2x) - 4\cos(2x) \quad f'''(x) = 8\sin(2x) - 8\cos(2x) +$

$f^{IV}(x) = 16\sin(2x) + 16\cos(2x) \quad f^{XI} = -2048\cos(2x) + 2048\sin(2x)$

5. $A = f'(1) \quad f'(x) = 2x(\ln 3)3^{x^2-1} \quad A = 2\ln 3$

$f^{-1}(x) = e^x + 1 \quad f^{-1}(1) = e+1 = B \quad C = 2 \quad f'(x) = \frac{2x}{\sqrt{1-(x^2-1)^2}}$

$D = f''(1) \quad f''(x) = 4x(2x) + 4(x^2-1) \quad f'(x) = 2x[2(x^2-1)] = 4x(x^2-1) \quad f''(1) = 8 = D$

$\frac{BC}{AD} = \frac{2(e+1)}{2\ln 3(8)} = \frac{e+1}{8\ln(3)}$

6. $y' = 3e^x + 2x - 2 \quad FOCUS = (0,2) \quad m_{tan} = 3+0-2 = 1 \quad x+y = 2 \quad x+y-2 = 0$

$m_{norm} = -1$

7. $f(x) = xe^x \quad f'(x) = xe^x + e^x \quad f''(x) = xe^x + e^x + e^x \quad f'''(x) = e^x + e^x(x+2)$

$f''(x) \; e^x(x+2) \quad f'''(x) = e^x(x+3)$

1st derivative decreases $(-\infty, -2)$ concave up $(-3, \infty)$ $\therefore (-3, -2)$

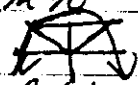
8. $\int_0^k \cos(2x)\,dx = \begin{matrix} u = 2x \\ du = 2dx \end{matrix} \frac{1}{2}\int_0^{2k} \cos u\,du = \frac{1}{2} \quad \sin 2k - \sin 0 = 0$

$2K = \frac{\pi}{2} \quad K = \frac{\pi}{4}$

$\frac{2\pi}{\frac{2\pi}{3}} = 3 \quad \frac{4\pi}{3} = r\pi \quad \frac{K}{r} = \frac{\pi}{4} \div \frac{4}{3} = \frac{\pi}{4} \cdot \frac{3}{4} = \frac{3\pi}{16}$

9. $y' = \frac{2\pi}{3}\sec\left[\frac{2\pi x}{3}\right]\tan\left[\frac{2\pi x}{3}\right] + 8x \quad b = \lim_{n\to 0} \frac{3\cos(3n) - 3e^n}{2n} =$

$\lim_{n\to 0} \frac{-9\sin(3n) - 3e^n}{2} = -\frac{3}{2} \quad \frac{2\pi}{3}\left[\sec(-\pi)\tan(-\pi)\right] + 8(-\frac{3}{2}) = 0 - 12 = -12$

10.  $A = \frac{1}{2}(2x, y) \quad A(x) = 36x - x^3 \quad x^2 = 12 \quad A = 2\sqrt{3}(36-12)$

$A(x) = x(36-x^2) \quad A'(x) = 36 - 3x^2 \quad x = 2\sqrt{12} \quad A = 48\sqrt{3}$

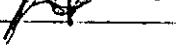
11. $\ln(y) = x\ln(\sin x) \quad \frac{y'}{y} = x\left[\frac{\cos x}{\sin x}\right] + \ln(\sin x) \quad y' = \left[x\cot x + \ln(\sin x)\right](\sin x)^x$

at $x = \frac{\pi}{2}$ $y' = \frac{\pi}{2}\cot\frac{\pi}{2} + \ln(\sin\frac{\pi}{2})(\sin\frac{\pi}{2}) = (0+0)1 = 0$ horizontal tangent.

$y = (1)^{\frac{\pi}{2}} = 1 \quad y = 1.$

12. $3\left(\frac{dy}{dx}\right) = e^x y \quad \int 3\left(\frac{dy}{y}\right) = \int e^x dx \quad 3\ln|y| = e^x + C \quad c = -1 \quad 3\ln e = e^x - 1$

$3\ln(1) = e^0 + C \quad 3\ln|y| = e^x - 1 \quad 4 = e^x \quad x = \ln 4$

13. $f'(x) = 6x^2 - 18x - 24 = 6(x^2 - 3x - 4) = 6(x-4)(x+1)$ 

$f(4) = -2 - 9 + 24 + 2 = 26 - 11 = 15$

14.  $A(x) = x^2 + \frac{\sqrt{3}x^2}{4} \quad x = 48 \quad dx = \frac{1}{2} \quad \frac{dy}{dx} = 2x + \frac{\sqrt{3}x}{2}$

$dy = \left[2(48) + 24\sqrt{3}\right] \cdot \frac{1}{2} = 8 + 2\sqrt{3}$

15. $\lim_{x\to 0} \frac{x - e^x + 1}{x(e^x - 1)} = \lim_{x\to 0} \frac{1 - e^x}{xe^x + e^x - 1} = \lim_{x\to 0} \left[\frac{-e^x}{xe^x + e^x + e^x}\right] = \frac{-1}{2}$